



## Evidence of Effective Early Numeracy Models

March 3, 2014

### INFORMATION REQUEST

This state was seeking information about the evidence that exists for effective early numeracy models. The questions posed were:

1. What does the research say about effective approaches to support early numeracy?
2. What additional resources are available that the state can use to support districts' use of evidence-based models and approaches?

### State's Goal

The information was needed by the state so it could support districts use of effective models and approaches to improve children's early literacy and numeracy outcomes.

### Background and Context

The Northeast Comprehensive Center and CELO met with the state education agency early childhood specialist to coordinate technical assistance to the state. During the meeting, the Northeast Comprehensive Center agreed to work with the early childhood specialist to convene stakeholders from each region throughout the state to articulate priority needs in the areas of early literacy and numeracy. CELO agreed to provide the group with information and resources regarding the evidence of effective early literacy and numeracy models and approaches.

### Response

CELO staff reviewed key research including information from the What Works Clearinghouse obtained by the Regional Education Lab and by CELO staff, and through a review research obtained by the Council of Chief State School Officers (CCSSO) and the National Institute for Early Education Research (NIEER).

### What We Know – Early Numeracy Interventions

According to NIEER authors Kimberly Brenneman, Judi Stevenson-Boyd and Ellen C. Frede, improving mathematics is of great concern to educators and policymakers. Because early experiences affect later education outcomes, providing young children with research-based mathematics learning opportunities is likely to pay off with increased achievement, literacy, and work skills in these critical areas. Their policy brief<sup>1</sup> and work by Clements and others has found that:

- Young children have foundational competence in mathematics before they begin formal schooling.

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<sup>1</sup> Brenneman, K., Stevenson-Boyd, J., and Frede, E.C. (2009). *Math and science in preschool: Policies and practice* (NIEER Policy Brief Issue 19). New Brunswick, NJ: National Institute for Early Education Research. Retrieved from : <http://nieer.org/resources/policybriefs/20.pdf>.



- Young children are motivated to explore mathematical concepts they encounter in their everyday interactions with the world.
- Despite the existence of learning standards and increased curricular attention to mathematics, they tend not to be emphasized by teacher preparation or in-service professional development programs and evidence suggests that preschool educators tend not to support mathematics and science learning.
- In general, little is known about effective teaching of mathematics in preschool.

Recently, researchers have come to a basic agreement on the nature of mathematical learning trajectories<sup>2</sup>. Learning trajectories have three parts: a) a mathematical goal; b) a developmental path along which children develop to reach that goal; and c) a set of instructional activities, or tasks, matched to each of the levels of thinking in that path that help children develop higher levels of thinking. Table 1 below summarizes the learning trajectories for early numeracy as reported by Clements and Samara<sup>3</sup>.

**Table 1. Samples from the Learning Trajectory for Counting<sup>4</sup>**

Age	Progression	Task
One	Pre-Counter: Verbal: No verbal counting. Chanter: Chants “sing-song” or sometimes-indistinguishable number words.	Associate number words with quantities and as components of the counting sequence.  Repeated experience with the counting sequence in varied contexts.
Two	Reciter: Verbal: Verbally counts with separate words, not necessarily in the correct order.	Provide repeated, frequent experience with the counting sequence in varied contexts. Count and Race: Children verbally count along with the computer (up to 50) by adding cars to a racetrack one at a time.
Three	Reciter: Verbal: Verbally counts to ten, with some correspondence with objects. Corresponder: Keeps one-to-one correspondence between counting words and objects (one word for each object), at least for small groups of	Count and Move: Have all children count from 1-10 or an appropriate number, making motions with each count. For example, say, “one” [touch head], “two” [touch shoulders], “three” [touch head], etc. At the computer, children click on objects one at a time while the numbers from one to ten

<sup>2</sup> Clements, D.H., & Sarama, J. (2004). Learning trajectories in mathematics education. *Mathematical Thinking and Learning* 6(2), 81-89. doi: 10.1207/s15327833mtl0602\_1

<sup>3</sup> Clements, D. H. & Sarama, J. (2009). Learning trajectories in early mathematics – sequences of acquisition and teaching. *Encyclopedia of Language and Literacy Development* (pp. 1-7). London, ON: Canadian Language and Literacy Research Network. Retrieved from: <http://literacyencyclopedia.ca/pdfs/topic.php?topid=270>

<sup>4</sup> Clements & Sarama, 2009



Age	Progression	Task
	objects laid in a line.	are counted aloud. For example, they click on pieces of food and a bite is taken out of each as it is counted.
Four	Counter (Small Numbers): Accurately counts objects in a line to 5 and answers the “how many” question with the last number counted.  Producer (Small Numbers)	Cubes in the Box: Have the child count as small set of cubes. Put them in the box and close the lid. Then ask the child how many cubes you are hiding. If the child is ready, have him/her write the numeral. Dump them out and count together to check. Pizza Pizzazz: Children count items up to 5, putting toppings on a pizza to match a target amount. Counts out objects to 5. Recognizes that counting is relevant to situations in which a certain number must be placed. Count Motions. While waiting during transitions, have children count how many times you jump or clap, or some other motion. Then have them do those motions the same number of times. Initially, count the actions with children.
Five	Counter and Producer (10+): Counts and counts out objects accurately to 10, then beyond (to about 30). Has explicit understanding of cardinality (how numbers tell how many). Keeps track of objects that have and have not been counted, even indifferent arrangements.	To allow children to count to 20 and beyond, have them make towers with other objects such as coins. Children build a tower as high as they can, placing more coins, but not straightening coins already in the tower. The goal is to estimate and then count to find out how many coins are in your tallest tower.

Clements argues that although learning trajectories have proven to be effective for early mathematics curricula and professional development<sup>5</sup>, there have been too few studies that have compared various ways of implementing them. They argue that while several learning trajectories are based on

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<sup>5</sup> Clemens, D.H., & Sarama, J. (2007a). Early childhood mathematics learning. In F.K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (461- 555). New York: Information Age Publishing; Clemens, D. H., & Sarama, J. (2007b). *SRA real math building blocks. Teacher's resource guide preK*. Columbus, OH: SRA/McGraw- Hill; Sarama, J., Clemens, D. H., Starkey, P., Klein, A., & Wakeley, A. (2008). Scaling up the implementation of a pre-kindergarten mathematics curriculum: Teaching for understanding with trajectories and technologies. *Journal of Research on Educational Effectiveness*, 1, 89-119.



considerable research, such as those for counting and arithmetic, others, such as patterning and measurement, have a smaller research base.

### **What Works Clearinghouse Findings**

The WWC was reviewed for early mathematics interventions. CEELo searched for evidence published in the past five years. Fewer reviews of mathematics interventions were included in the WWC than literacy interventions and many of the reviews indicated that currently many interventions lack rigorous evaluations. Nonetheless, the following interventions were found to have positive impact on students' early numeracy outcomes:

- **Building Blocks for Math** was found to have positive effects on mathematics achievement. SRA Real Math Building Blocks PreK (also referred to as Building Blocks for Math) is a supplemental mathematics curriculum designed to develop preschool children's early mathematical knowledge through various individual and small- and large-group activities. It uses Building Blocks for Math PreK software, manipulatives, and print material. Building Blocks for Math embeds mathematical learning in children's daily activities, ranging from designated math activities to circle and story time, with the goal of helping children relate their informal math knowledge to more formal mathematical concepts. [http://ies.ed.gov/ncee/wwc/pdf/intervention\\_reports/WWC\\_Building\\_Blocks\\_072307.pdf](http://ies.ed.gov/ncee/wwc/pdf/intervention_reports/WWC_Building_Blocks_072307.pdf)
- **enVisionMath** was found to have potentially positive effects on mathematics achievement for elementary school students. enVisionMATH, published by Pearson Education, Inc., is a core curriculum for students in kindergarten through grade 6. The program seeks to help students develop an understanding of math concepts through problem-based instruction, small-group interaction, and visual learning with a focus on reasoning and modeling. Differentiated instruction and ongoing assessment are used to meet the needs of students at all ability levels. [http://ies.ed.gov/ncee/wwc/pdf/intervention\\_reports/wwc\\_envisionmath\\_011513.pdf](http://ies.ed.gov/ncee/wwc/pdf/intervention_reports/wwc_envisionmath_011513.pdf)
- **Investigations in Number, Data, and Space<sup>®</sup>** was found to have potentially positive effects on mathematics achievement for elementary school students. Investigations in Number, Data, and Space<sup>®</sup>, published by Pearson Scott Foresman, is an activity-based K–5 mathematics curriculum designed to help students understand number and operations, geometry, data, measurement, and early algebra. Each instructional unit focuses on a particular content area and lasts for two to five-and-a-half weeks. The curriculum encourages students to develop their own strategies for solving problems and engage in discussion about their reasoning and ideas. Students work in a variety of groupings, including individually, pairs, small groups, and whole class.
- **Pre-K Mathematics with DLM Early Childhood Express Math** was found to have no discernible effects on oral language, print knowledge, and phonological processing but positive effects on math for preschool children. Pre-K Mathematics is a supplemental curriculum designed to develop the



informal mathematical knowledge and skills of preschool children with content organized into seven units. Specific math concepts and skills from each unit are taught in the classroom through teacher-guided, small-group activities with concrete manipulatives. The curriculum also includes take-home activities that parallel the small-group classroom activities, and are designed to help parents support their children's mathematical development at home. The DLM Early Childhood Express Math software includes corresponding math-based activities to reinforce math concepts taught in the classroom. [http://ies.ed.gov/ncee/wwc/pdf/intervention\\_reports/wwc\\_prekmath\\_121713.pdf](http://ies.ed.gov/ncee/wwc/pdf/intervention_reports/wwc_prekmath_121713.pdf)

- **Saxon Math** was found to have potentially positive effects on mathematics achievement for elementary school students. Saxon Math, published by Houghton Mifflin Harcourt, is a core curriculum for students in grades K–5. New concepts are introduced gradually and integrated with previously introduced content so that concepts are developed, reviewed, and practiced over time rather than being taught during discrete periods of time, such as in chapters or units. Instruction is built around math conversations that engage students in learning, as well as continuous practice with hands-on activities, manipulatives, and paper-pencil methods. The program includes frequent, cumulative assessments used to direct targeted remediation and support to struggling students. Starting in grade 3, the focus shifts from teacher-directed instruction to a more student-directed, independent learning approach, though math conversations continue to be used to introduce new concepts. [http://ies.ed.gov/ncee/wwc/pdf/intervention\\_reports/wwc\\_saxon\\_052913.pdf](http://ies.ed.gov/ncee/wwc/pdf/intervention_reports/wwc_saxon_052913.pdf)

No evidence that meets the WWC standards has yet been produced on the effectiveness of the following interventions on early numeracy:

- *Bright Beginnings* was found to have no discernible effects on oral language, print knowledge, phonological processing, or math for preschool children. Bright Beginnings is an early childhood curriculum, based in part on the High/Scope® and The Creative Curriculum® models, with additional emphasis on building early language and literacy skills.
- *The Creative Curriculum® for Preschool*, Fourth Edition, was found to have no discernible effects on oral language, print knowledge, phonological processing, or math for preschool children. The Creative Curriculum® for Preschool, Fourth Edition, is an early childhood curriculum that focuses on project-based investigations as a means for children to apply skills and addresses four areas of development: social/emotional, physical, cognitive, and language.
- *DreamBox Learning* was found to have no discernible effects on mathematics achievement for elementary school students. DreamBox Learning is a supplemental online mathematics program that provides adaptive instruction for students in grades K–5 and focuses on number and operations, place value, and number sense. The program aims to individualize instruction for each student with millions of unique paths through the curriculum intended to match each student's level



of comprehension and learning style. The curriculum is based on the National Council of Teachers of Mathematics (NCTM) standards.

- *Doors to Discovery™* was found to have no discernible effects on math for preschool children but did demonstrate some positive effects on some early literacy skills. *Doors to Discovery™* is a preschool literacy curriculum that uses eight thematic units of activities to help children build fundamental early literacy skills in oral language, phonological awareness, concepts of print, alphabet knowledge, writing, and comprehension.
- *Ladders to Literacy* was found to have no outcomes. *Ladders to Literacy* is a supplemental early literacy curriculum composed of 60 activities designed to develop children's print/book awareness, metalinguistic awareness, and oral language skills.

### Teacher and Teacher Leader Guides

Three practice guides were published in the WWC Clearinghouse on mathematics.

- **Five Evidence-Based Recommendations for Teaching Math to Young Children.** Summary: This practice guide provides an overview of the guide's five practical, evidence-based recommendations, examples of innovative tools and strategies teachers can use in their classrooms, and a brief discussion of the evidence that supports the guide's recommendation. The practice guide includes a selection of examples, figures, tables, and other features teachers can put to use in their classrooms right away.  
[http://ies.ed.gov/ncee/Wwc/pdf/practice\\_guides/wwc\\_empg\\_numbers\\_020714.pdf](http://ies.ed.gov/ncee/Wwc/pdf/practice_guides/wwc_empg_numbers_020714.pdf)
- **Developing Effective Fractions Instruction for Kindergarten Through 8th Grade.** Summary: This practice guide presents five recommendations intended to help educators improve students' understanding of fractions. Recommendations include strategies to develop young children's understanding of early fraction concepts and ideas for helping older children understand the meaning of fractions and the computations involved. The guide also highlights ways to build on students' existing strategies to solve problems involving ratios, rates, and proportions. The guide recommends the following: 1) Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts; 2) Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward; 3) Help students understand why procedures for computations with fractions make sense; 4) Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems; 5) Professional development programs should place a high priority on improving teachers' understanding of fractions and of how to teach them. [http://ies.ed.gov/ncee/wwc/pdf/practice\\_guides/fractions\\_pg\\_093010.pdf](http://ies.ed.gov/ncee/wwc/pdf/practice_guides/fractions_pg_093010.pdf)



- **Teaching Math to Young Children, November 2013.** Summary: This practice guide provides five recommendations for teaching math to children in preschool, prekindergarten, and kindergarten. Each recommendation includes implementation steps and solutions for common roadblocks. The recommendations also summarize and rate supporting evidence. The guide is geared toward teachers, administrators, and other educators who want to build a strong foundation for later math learning. The following recommendations are included: 1) Teach number and operations using a developmental progression; 2) Teach geometry, patterns, measurement, and data analysis using a developmental progression; 3) Use progress monitoring to ensure that math instruction builds on what each child knows; 4) Teach children to view and describe their world mathematically; 5) Dedicate time each day to teaching math, and integrate math instruction throughout the school day. [http://ies.ed.gov/ncee/wwc/pdf/practice\\_guides/early\\_math\\_pg\\_111313.pdf](http://ies.ed.gov/ncee/wwc/pdf/practice_guides/early_math_pg_111313.pdf)



## Resources and Links

The following organizations have additional resources on early numeracy.

- National Council of Teachers of Mathematics: <http://www.nctm.org/resources/content.aspx?id=9300>

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