

## Mathematics Standards Introduction

---

A strong mathematics education depends upon a clear understanding of its interrelated concepts, skills and practices to ensure students are on the pathway to success in their academic careers. The knowledge and skills students need to be prepared for mathematics in college, career, and life are woven throughout the K-12 mathematics performance expectations.

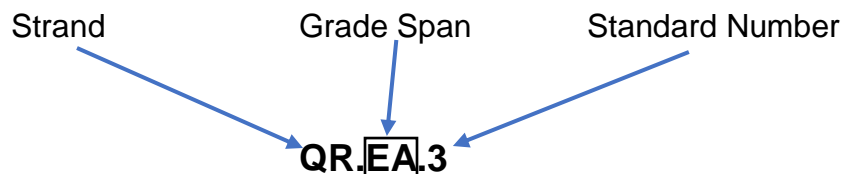
### Outline of Mathematics Strands and Standards

These mathematical performance expectations are building blocks to standards. The standards are grouped into four strands:

- **Quantitative Reasoning** (Blue): Counting and Cardinality, Number and Operations in Base Ten, Number and Operations Fractions, Ratio and Proportional Relations, The Number System, and Number and Quantity.
- **Algebraic Reasoning** (Green): Operations and Algebraic Thinking, Expressions and Equations, Functions, and Algebra
- **Geometric Reasoning** (Red): Geometry
- **Statistical Reasoning** (Purple): Measurement and Data, Statistics and Probability

These mathematical performance expectations are broken into three grade spans: **Childhood** (K-5), **Early Adolescence** (6-8), and **Adolescence** (9-Diploma). The strands are color-coded, as indicated above, for continuity throughout the grade spans. Standards do not work in isolation, they are connected through and across strands.

### How to Read the Standards



Within the high school performance expectations, modeling is woven throughout the four strands and is denoted with a star (★). The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.

## The Guiding Principles & Standards for Mathematical Practice

---

The Guiding Principles influence education in Maine and should be reflected throughout Mathematics curriculum. The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Full descriptions of the Guiding Principles and Standards for Mathematical Practice can be found in the Supplemental Material. Examples of how students can show evidence of those **Guiding Principles** and **Standards for Mathematical Practice** may include:

### Guiding Principles

---

- A. **A clear and effective communicator:** Students will use written, oral, symbolic, and visual forms of expression to communicate mathematically.
- B. **A self-directed and lifelong learner:** Students generate and persevere in solving questions while demonstrating a growth mindset.
- C. **A creative and practical problem solver:** Students will pose and solve mathematical problems by using a variety of strategies that connect to real-world examples.
- D. **A responsible and involved citizen:** Students make sense of the world around them through mathematics including economic literacy.
- E. **An integrative and informed thinker:** Students connect mathematics to other learning by understanding the interrelationships of mathematical ideas and the role math plays in other disciplines and life.

### Standards for Mathematical Practice

---

- 1. **Make sense of problems and persevere in solving them:** Students will plan strategies to use and persevere in solving math problems.
- 2. **Reason abstractly and quantitatively:** Students will think about numbers in many ways and make sense of numerical relationships as they solve problems.
- 3. **Construct viable arguments and critique the reasoning of others:** Students will explain their thinking and make sense of the thinking of others.
- 4. **Model with mathematics:** Students will use representations to show their thinking in a variety of ways.
- 5. **Use appropriate tools strategically:** Students will use math tools such as tables, diagrams, and technology to explore and deepen their understanding of concepts.
- 6. **Attend to precision:** Students will use precise mathematical language and check their work for accuracy.
- 7. **Look for and make use of structure:** Students will use their current mathematical understandings to identify patterns and structure to make sense of new learning.
- 8. **Look for and express regularity in repeated reasoning:** Students will look for patterns and rules to help create general methods and shortcuts that can be applied to similar mathematical problems.

## Statistical Reasoning

---

Statistical reasoning is the way people analyze data and make sense of information. It involves generalizations that connect one concept to another. In this K-5 strand, students will develop strategies to represent and interpret data, describe and compare measurable attributes, and understand concepts of measurement including perimeter, area, volume, time, and money.

Students in grades 6-8 continue to develop their ability to think statistically. Measures of central tendency (mean, median, and mode) as well as measures of variability (range, interquartile range, mean absolute deviation) are used to describe data. Previous work with single data distributions is expanded to compare two data distributions and address questions about differences between populations. Informal work with random sampling and learning about the importance of representative samples for drawing inferences is introduced. Students then expand their statistical understanding to include connections involving modeling with linear equations, as well as non-linear expressions. Looking for patterns in a bivariate data system is emphasized.

In grades 9-12 students extend their statistical understanding of univariate and bi-variate data in a real-world context. This understanding is used to make decisions or predictions based on the data. Since data can be variable, statistics provide the tools for taking this variability into account. Data can be categorical or quantitative in nature. Appropriate methods for collecting, displaying, summarizing, and analyzing data are learned and employed. Algebraic and geometric reasoning are utilized to create linear regression models in order to interpret the relationship between two quantitative variables when appropriate.

The conditions under which data are collected and the use of randomization in the design of a study are necessary for drawing valid conclusions about the population under study. Since random processes can be described mathematically by using a probability model, the role of probability in making predictions or in making decisions becomes evident. Technology makes it possible to generate plots, find regression functions, compute correlation coefficients, and run simulations to better understand data. Statistical reasoning is a deeply rich and complex process which is essential to comprehend in order to stay informed in civic matters and personal decision-making.

Strand	Statistical Reasoning - Measurement & Data		
Standard	SR.C.1 Describe and compare measurable attributes.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations	<p><b>K.MD.A.1:</b> Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.</p> <p><b>K.MD.A.2:</b> Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute and describe the difference. <i>For example, directly compare the heights of two children and describe one child as taller/shorter.</i></p>	<p><b>1.MD.A.1:</b> Order three objects by length; compare the lengths of two objects indirectly by using a third object.</p> <p><b>1.MD.A.2:</b> Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. <i>Limit to contexts where the object being measured is spanned by a</i></p>	<p><b>2.MD.A.4:</b> Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.</p> <p><b>2.MD.A.1:</b> Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.</p> <p><b>2.MD.A.2:</b> Measure the length of an object twice, using length units of different lengths for the two measurements; describe how</p>

		<i>whole number of length units with no gaps or overlaps.</i>	the two measurements relate to the size of the unit chosen.  <b>2.MD.A.3:</b> Estimate lengths using units of inches, feet, centimeters, and meters.
	<b>K.MD.B.3:</b> Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.)		
Strand	<b>Statistical Reasoning - Measurement &amp; Data</b>		
Standard	<b>SR.C.2</b> Represent and interpret data.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations		<b>1.MD.C.4:</b> Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	<b>2.MD.D.9:</b> Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Organize and record data on a line plot, where the horizontal scale is marked off in whole-number units.

			<b>2.MD.D.10:</b> Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.
Strand	<b>Statistical Reasoning - Measurement &amp; Data</b>		
Standard	<b>SR.C.3</b> Relate addition and subtraction to length.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations			<p><b>2.MD.B.5:</b> Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.</p> <p><b>2.MD.B.6:</b> Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and</p>

			differences within 100 on a number line diagram.
Strand	<b>Statistical Reasoning - Measurement &amp; Data</b>		
Standard	<b>SR.C.4</b> Work with time and money.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations		<p><b>1.MD.B.3:</b> Tell and write time in hours and half-hours using analog and digital clocks.</p> <p><b>1.MD.D.5:</b> Identify the coins and each corresponding value. (e.g. penny, nickel, dime, and quarter)</p>	<p><b>2.MD.C.7:</b> Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.</p> <p><b>2.MD.C.8:</b> Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?</p>
Strand	<b>Statistical Reasoning - Measurement &amp; Data</b>		
Standard	<b>SR.C.5</b> Solve problems involving measurement, conversion of measurement and estimation of intervals of time, liquid volumes, and masses of objects.		
	Childhood		
	Grade 3	Grade 4	Grade 5
	<p>Excludes compound units such as cm<sup>3</sup> and finding the geometric volume of a container</p> <p>Excludes multiplicative comparison problems (problems involving notions of "times as much")</p>		
Performance Expectations	<b>3.MD.A.1:</b> Tell and write time to the nearest minute and measure time intervals in minutes using analog and digital clocks. Solve		

	<p>word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.</p> <p><b>3.MD.A.2:</b> Measure and estimate liquid volumes and masses of objects using standard metric units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same metric units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.</p>	<p><b>4.MD.A.1:</b> Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. <i>For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft</i></p>	<p><b>5.MD.A.1:</b> Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.</p>
--	---	---	--



		<p><i>snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...</i></p> <p><b>4.MD.A.2:</b> Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.</p>	
Strand	<b>Statistical Reasoning - Measurement &amp; Data</b>		
Standard	<b>SR.C.6</b> Represent and interpret data.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations	<p><b>3.MD.B.3:</b> Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. <i>For example, draw a bar graph in which</i></p>		

	<p><i>each square in the bar graph might represent 5 pets.</i></p> <p><b>3.MD.B.4:</b> Generate measurement data by measuring lengths of objects using rulers marked with halves and fourths of an inch. Record and show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or fourths.</p>	<p><b>4.MD.B.4:</b> Make a line plot to display a data set of measurements in fractions of a unit (<math>\frac{1}{2}</math>, <math>\frac{1}{4}</math>, <math>\frac{1}{8}</math>). Solve problems involving addition and subtraction of fractions by using information presented in line plots. <i>For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.</i></p>	<p><b>5.MD.B.2:</b> Make a line plot to display a data set of measurements in fractions of a unit (<math>\frac{1}{2}</math>, <math>\frac{1}{4}</math>, <math>\frac{1}{8}</math>). Use operations on fractions for this grade to solve problems involving information presented in line plots. <i>For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.</i></p>
Strand	<b>Statistical Reasoning - Measurement &amp; Data</b>		
Standard	<b>SR.C.7</b> Understand concepts of Geometric measurement: involving perimeter, area, and volume.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations	<p><b>3.MD.C.5:</b> Recognize area as an attribute of plane figures and understand concepts of area measurement.</p> <p><b>3.MD.C.5a:</b> A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area,</p>		<p><b>5.MD.C.3:</b> Recognize volume as an attribute of solid figures and understand concepts of volume measurement.</p> <p><b>5.MD.C.3a:</b> A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of</p>

	<p>and can be used to measure area.</p> <p><b>3.MD.C.5b:</b> A plane figure which can be covered without gaps or overlaps by <math>n</math> unit squares is said to have an area of <math>n</math> square units.</p> <p><b>3.MD.C.6:</b> Measure areas by counting unit squares (square cm, square m, square in, square ft, and non- standard units</p> <p><b>3.MD.C.7:</b> Relate area to the operations of multiplication and addition.</p> <p><b>3.MD.C.7a:</b> Find the area of a rectangle with whole-number side lengths by tiling it and show that the area is the same as would be found by multiplying the side lengths.</p> <p><b>3.MD.C.7b:</b> Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems and represent whole-number products as rectangular</p>		<p>volume, and can be used to measure volume.</p> <p><b>5.MD.C.3b:</b> A solid figure which can be packed without gaps or overlaps using <math>n</math> unit cubes is said to have a volume of <math>n</math> cubic units.</p> <p><b>5.MD.C.4:</b> Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and non-standard units.</p> <p><b>5.MD.C.5:</b> Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. When finding volumes of objects answers will be in cubic units.</p> <p><b>5.MD.C.5a:</b> Find the volume of a right rectangular prism with whole-number edge lengths by packing it with unit cubes and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by</p>
--	---	--	---

	<p>areas in mathematical reasoning.</p> <p><b>3.MD.C.7c:</b> Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths <math>a</math> and <math>b + c</math> is the sum of <math>a \times b</math> and <math>a \times c</math>. Use area models to represent the distributive property in mathematical reasoning.</p> <p><b>3.MD.C.7d:</b> Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.</p> <p><b>3.MD.D.8:</b> Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.</p>	<p><b>4.MD.A.3:</b> Apply the area and perimeter formulas for rectangles in real world and mathematical problems. <i>For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.</i></p>	<p>multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.</p> <p><b>5.MD.C.5b:</b> Apply the formulas <math>V = l \times w \times h</math> and <math>V = B \times h</math> (where <math>B</math> stands for the area of the base) for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.</p> <p><b>5.MD.C.5c:</b> Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.</p>
--	--	--	---

Strand	<b>Statistical Reasoning - Measurement &amp; Data</b>		
Standard	<b>SR.C.8</b> Geometric measurement: understand concept of angle and measure angles.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations		<p><b>4.MD.C.5:</b> Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:</p> <p><b>4.Md.C.5a:</b> An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through <math>\frac{1}{360}</math> of a circle is called a "one-degree angle," and can be used to measure angles.</p> <p><b>4.MD.C.5b:</b> An angle that turns through <math>n</math> one-degree angles is said to have an angle measure of <math>n</math> degrees.</p> <p><b>4.MD.C.6:</b> Measure angles in whole-number degrees using a</p>	

		<p>protractor. Sketch angles of specified measure.</p> <p><b>4.MD.C.7:</b> Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.</p>	
--	--	--	--

Strand	<b>Statistical Reasoning - Statistics &amp; Probability</b>
Standard	<b>SR.EA.1</b> Summarize distribution using measures of center, variability, and graphical displays.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p><b>6.SP.A.1:</b> Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. <i>For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.</i></p> <p><b>6.SP.A.2:</b> Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center (mean, median and/or mode), spread (range and/or interquartile range), and overall shape.</p> <p><b>6.SP.A.3:</b> Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.</p>

	<p><b>6.SP.B.4:</b> Display numerical data in plots on a number line, including dot plots, histograms, and box plots.</p> <p><b>6.SP.B.5:</b> Summarize numerical data sets in relation to their context, such as by:</p> <p><b>6.SP.B.5a:</b> Reporting the number of observations.</p> <p><b>6.SP.B.5b:</b> Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.</p> <p><b>6.SP.B.5c:</b> Calculating quantitative measures of center (median and/or mean) and variability (range and/or interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.</p> <p><b>6.SP.B.5d:</b> Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.</p>
Strand	<b>Statistical Reasoning - Statistics &amp; Probability</b>
Standard	<b>SR.EA.2</b> Use random sampling, visual representations, and measures of center and variability to draw inferences about one or more populations.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p><b>7.SP.A.1:</b> Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.</p> <p><b>7.SP.A.2:</b> Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. <i>For example, estimate the mean length of a largemouth bass in a lake by randomly sampling largemouth bass from the lake; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.</i></p> <p><b>7.SP.B.3:</b> Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. <i>For example, the mean height of players on the basketball team is 10 cm greater than the mean</i></p>

	<p><i>height of players on the soccer team and both distributions have similar variability (mean absolute deviation) of about 5 cm. The difference between the mean heights of the two teams (10 cm) is about twice the variability (5 cm mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.</i></p> <p><b>7.SP.B.4:</b> Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. <i>For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.</i></p>
Strand	<b>Statistical Reasoning - Statistics &amp; Probability</b>
Standard	<b>SR.EA.3</b> Investigate chance processes and develop, use, and evaluate probability models.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p><b>7.SP.C.5:</b> Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.</p> <p><b>7.SP.C.6:</b> Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. <i>For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.</i></p> <p><b>7.SP.C.7:</b> Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.</p> <p><b>7.SP.C.7a:</b> Develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events. <i>For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.</i></p> <p><b>7.SP.C.7b:</b> Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. <i>For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?</i></p>



	<p><b>7.SP.C.8:</b> Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.</p> <p><b>7.SP.C.8a:</b> Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.</p> <p><b>7.SP.C.8b:</b> Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.</p> <p><b>7.SP.C.8c:</b> Design and use a simulation to generate frequencies for compound events. <i>For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?</i></p>
Strand	<b>Statistical Reasoning - Statistics &amp; Probability</b>
Standard	<b>SR.EA.4</b> Investigate patterns of association in bivariate data.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p><b>8.SP.A.1:</b> Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.</p> <p><b>8.SP.A.2:</b> Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.</p> <p><b>8.SP.A.3:</b> Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. <i>For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.</i></p> <p><b>8.SP.A.4:</b> Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. <i>For example, collect data from</i></p>

	<i>students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?</i>
Strand	<p align="center"><b>Statistical Reasoning - Statistics &amp; Probability: Interpreting Categorical &amp; Quantitative Data</b></p> <p>Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).</p>
Standard	<b>SR.A.1</b> Summarize, represent, and interpret data on a single count or measurement variable. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	<p><b>HSS.ID.A.1:</b> Represent data with plots on the real number line (dot plots, histograms, and box plots). ★</p> <p><b>HSS.ID.A.2:</b> Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. ★</p> <p><b>HSS.ID.A.3:</b> Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★</p> <p><b>HSS.ID.A.4:</b> Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. ★</p>
Strand	<p align="center"><b>Statistical Reasoning - Statistics &amp; Probability: Interpreting Categorical &amp; Quantitative Data</b></p> <p>Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).</p>
Standard	<b>SR.A.2</b> Summarize, represent, and interpret data on two categorical variables and two quantitative variables. ★

	Adolescence Grades 9-Diploma
Performance Expectations	<p><b>HSS.ID.B.5:</b> Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. ★</p> <p><b>HSS.ID.B.6:</b> Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ★</p> <p><b>HSS.ID.B.6a:</b> Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. ★</p> <p><b>HSS.ID.B.6b:</b> Informally assess the fit of a function by plotting and analyzing residuals. ★</p> <p><b>HSS.ID.B.6c:</b> Fit a linear function for a scatter plot that suggests a linear association. ★</p>
Strand	<p align="center"><b>Statistical Reasoning - Statistics &amp; Probability: Interpreting Categorical &amp; Quantitative Data</b></p> <p><small>Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).</small></p>
Standard	<b>SR.A.3</b> Interpret linear models. ★
	Adolescence Grades 9-Diploma
Performance Expectations	<p><b>HSS.ID.C.7:</b> Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ★</p> <p><b>HSS.ID.C.8:</b> Compute (using technology) and interpret the correlation coefficient of a linear fit. ★</p> <p><b>HSS.ID.C.9:</b> Distinguish between correlation and causation. ★</p>
Strand	<p align="center"><b>Statistical Reasoning - Statistics &amp; Probability: Making Inferences &amp; Justifying Conclusions</b></p>

	<p>Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).</p>
Standard	<b>SR.A.4</b> Understand and evaluate random processes underlying statistical experiments. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	<p><b>HSS.IC.A.1:</b> Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ★</p> <p><b>HSS.IC.A.2:</b> Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. <i>For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?</i> ★</p>
Strand	<p><b>Statistical Reasoning - Statistics &amp; Probability: Making Inferences &amp; Justifying Conclusions</b></p> <p>Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).</p>
Standard	<b>SR.A.5</b> Make inferences and justify conclusions from sample surveys, experiments, and observational studies. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	<p><b>HSS.IC.B.3:</b> Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. ★</p> <p><b>HSS.IC.B.4:</b> Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. ★</p> <p><b>HSS.IC.B.5:</b> Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. ★</p>

	<b>HSS.IC.B.6:</b> Evaluate reports based on data. <i>For example, use an article in the local news and interpret the validity of the information presented. Consider animal wildlife reports, medical studies, and/or manufacturer claims.</i> ★
Strand	<b>Statistical Reasoning - Statistics &amp; Probability: Conditional Probability &amp; the Rules of Probability</b>  Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).
Standard	<b>SR.A.6</b> Understand independence and conditional probability and use them to interpret data. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	<p><b>HSS.CP.A.1:</b> Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). ★</p> <p><b>HSS.CP.A.2:</b> Understand that two events <math>A</math> and <math>B</math> are independent if the probability of <math>A</math> and <math>B</math> occurring together is the product of their probabilities, and use this characterization to determine if they are independent. ★</p> <p><b>HSS.CP.A.3:</b> Understand the conditional probability of <math>A</math> given <math>B</math> as <math>P(A \text{ and } B)/P(B)</math>, and interpret independence of <math>A</math> and <math>B</math> as saying that the conditional probability of <math>A</math> given <math>B</math> is the same as the probability of <math>A</math>, and the conditional probability of <math>B</math> given <math>A</math> is the same as the probability of <math>B</math>. ★</p> <p><b>HSS.CP.A.4:</b> Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. <i>For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.</i> ★</p>

	<p><b>HSS.CP.A.5:</b> Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. <i>For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</i> ★</p>
Strand	<p style="text-align: center;"><b>Statistical Reasoning - Statistics &amp; Probability: Conditional Probability &amp; the Rules of Probability</b></p> <p><b>Modeling Standards:</b> Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).</p> <p>The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.</p>
Standard	<p><b>SR.A.7</b> Use the rules of probability to compute probabilities of compound events in a uniform probability model. ★</p>
	Adolescence
	Grades 9-Diploma
Performance Expectations	<p><b>HSS.CP.B.6:</b> Find the conditional probability of <math>A</math> given <math>B</math> as the fraction of <math>B</math>'s outcomes that also belong to <math>A</math>, and interpret the answer in terms of the model. ★</p> <p><b>HSS.CP.B.7:</b> Apply the Addition Rule, <math>P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)</math>, and interpret the answer in terms of the model. ★</p> <p><b>(+) HSS.CP.B.8:</b> Apply the general Multiplication Rule in a uniform probability model, <math>P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)</math>, and interpret the answer in terms of the model. ★</p> <p><b>(+) HSS.CP.B.9:</b> Use permutations and combinations to compute probabilities of compound events and solve problems. ★</p>
Strand	<p style="text-align: center;"><b>Statistical Reasoning - Statistics &amp; Probability: Using Probability to Make Decisions</b></p> <p><b>Modeling Standards:</b> Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).</p>

	The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.
Standard	<b>SR.A.8 (+)</b> Calculate expected values and use them to solve problems. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	<p><b>(+) HSS.MD.A.1:</b> Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. ★</p> <p><b>(+) HSS.MD.A.2:</b> Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. ★</p> <p><b>(+) HSS.MD.A.3:</b> Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. <i>For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.</i> ★</p> <p><b>(+) HSS.MD.A.4:</b> Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. <i>For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?</i> ★</p>
Strand	<p align="center"><b>Statistical Reasoning - Statistics &amp; Probability: Using Probability to Make Decisions</b></p> <p>Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).</p> <p>The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.</p>

Standard	<b>SR.A.9 (+) Use probability to evaluate outcomes of decisions. ★</b>
	Adolescence
	Grades 9-Diploma
Performance Expectations	<p><b>(+) HSS.MD.B.5:</b> Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. ★</p> <p><b>(+) HSS.MD.B.5a:</b> Find the expected payoff for a game of chance. <i>For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.</i> ★</p> <p><b>(+) HSS.MD.B.5b:</b> Evaluate and compare strategies on the basis of expected values. <i>For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.</i> ★</p> <p><b>(+) HSS.MD.B.6:</b> Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). ★</p> <p><b>(+) HSS.MD.B.7:</b> Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game and replacing the goalie with an extra skater). ★</p>

**Definitions:**

Strand: A body of knowledge in a content area identified by a simple title.

Standard: Enduring understandings and skills that students can apply and transfer to contexts that are new to the student.

Performance Expectation: Building blocks to the standard and measurable articulations of what the student understands and can do.