

## Mathematics Standards Introduction

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A strong mathematics education depends upon a clear understanding of its interrelated concepts, skills and practices to ensure students are on the pathway to success in their academic careers. The knowledge and skills students need to be prepared for mathematics in college, career, and life are woven throughout the K-12 mathematics performance expectations.

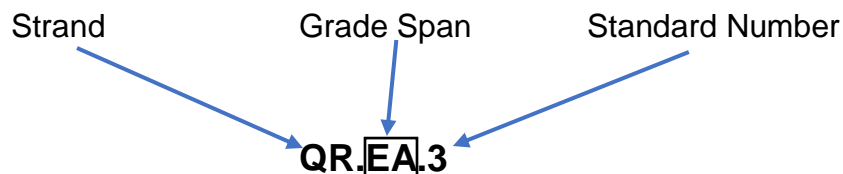
### Outline of Mathematics Strands and Standards

These mathematical performance expectations are building blocks to standards. The standards are grouped into four strands:

- **Quantitative Reasoning** (Blue): Counting and Cardinality, Number and Operations in Base Ten, Number and Operations Fractions, Ratio and Proportional Relations, The Number System, and Number and Quantity.
- **Algebraic Reasoning** (Green): Operations and Algebraic Thinking, Expressions and Equations, Functions, and Algebra
- **Geometric Reasoning** (Red): Geometry
- **Statistical Reasoning** (Purple): Measurement and Data, Statistics and Probability

These mathematical performance expectations are broken into three grade spans: **Childhood** (K-5), **Early Adolescence** (6-8), and **Adolescence** (9-Diploma). The strands are color-coded, as indicated above, for continuity throughout the grade spans. Standards do not work in isolation, they are connected through and across strands.

### How to Read the Standards



Within the high school performance expectations, modeling is woven throughout the four strands and is denoted with a star (★). The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.

## The Guiding Principles & Standards for Mathematical Practice

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The Guiding Principles influence education in Maine and should be reflected throughout Mathematics curriculum. The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Full descriptions of the Guiding Principles and Standards for Mathematical Practice can be found in the Supplemental Material. Examples of how students can show evidence of those **Guiding Principles** and **Standards for Mathematical Practice** may include:

### Guiding Principles

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- A. **A clear and effective communicator:** Students will use written, oral, symbolic, and visual forms of expression to communicate mathematically.
- B. **A self-directed and lifelong learner:** Students generate and persevere in solving questions while demonstrating a growth mindset.
- C. **A creative and practical problem solver:** Students will pose and solve mathematical problems by using a variety of strategies that connect to real-world examples.
- D. **A responsible and involved citizen:** Students make sense of the world around them through mathematics including economic literacy.
- E. **An integrative and informed thinker:** Students connect mathematics to other learning by understanding the interrelationships of mathematical ideas and the role math plays in other disciplines and life.

### Standards for Mathematical Practice

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- 1. **Make sense of problems and persevere in solving them:** Students will plan strategies to use and persevere in solving math problems.
- 2. **Reason abstractly and quantitatively:** Students will think about numbers in many ways and make sense of numerical relationships as they solve problems.
- 3. **Construct viable arguments and critique the reasoning of others:** Students will explain their thinking and make sense of the thinking of others.
- 4. **Model with mathematics:** Students will use representations to show their thinking in a variety of ways.
- 5. **Use appropriate tools strategically:** Students will use math tools such as tables, diagrams, and technology to explore and deepen their understanding of concepts.
- 6. **Attend to precision:** Students will use precise mathematical language and check their work for accuracy.
- 7. **Look for and make use of structure:** Students will use their current mathematical understandings to identify patterns and structure to make sense of new learning.
- 8. **Look for and express regularity in repeated reasoning:** Students will look for patterns and rules to help create general methods and shortcuts that can be applied to similar mathematical problems.

## Quantitative Reasoning

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Quantitative reasoning is the application of basic mathematics skills to analyze and process real-world information. In the K-5 grades, students use numbers, including written numerals, to represent quantities and to solve quantitative problems. Students will work on counting and cardinality, number and operations in Base Ten and fractions. Students will develop strategies to extend their understanding of the base ten system and apply those strategies to solve real-world problems using all four operations. Students progress from working with whole numbers to fractions and decimals.

In grades 6-8 students use reasoning about multiplication and division to solve ratio and rate problems about quantities. They develop an understanding of proportionality to solve problems and graph relationships. Overall, students extend and develop their understanding of rational numbers and can compute in all operations. Students use these operations to solve real-world problems. Students use this understanding of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

In the high school grades, the foundational concepts of operations with rational numbers and numerical properties built in the K-5 and 6-8 grade spans are applied to irrational numbers. Using a wider variety of units in modeling, (e.g. acceleration, currency conversions, and derived quantities such as person-hours and heating degree days), as well as the properties of rational and irrational numbers students are guided to the solution(s) to multi-step problems. Extending the properties of integer exponents to rational exponents deepens student understanding of how various but equivalent notations can facilitate their algebraic reasoning and problem-solving processes. Students are encouraged to expand these operations and properties into complex numbers, vectors, and matrices to further deepen their understanding of quantitative reasoning.

Strand	<b>Quantitative Reasoning- Counting and Cardinality</b>		
Standard	<b>QR.C.1</b> Know the number names and the count sequence.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations	<p><b>K.CC.A.1:</b> Count to 100 by ones and by tens</p> <p><b>K.CC.A.2:</b> Count forward beginning from a given number within the known sequence (instead of having to begin at 1)</p> <p><b>K.CC.A.3:</b> Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects)</p>		
Strand	<b>Quantitative Reasoning- Counting and Cardinality</b>		
Standard	<b>QR.C.2</b> Count to tell the number of objects.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations	<p><b>K.CC.B.4:</b> Understand the relationship between numbers and quantities; connect counting to cardinality.</p> <p><b>K.CC.B.4a:</b> When counting objects, say the number names in the standard order, pairing each object with one and only one number name</p>		

	<p>and each number name with one and only one object.</p> <p><b>K.CC.B.4b:</b> Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.</p> <p><b>K.CC.B.4c:</b> Understand that each successive number name refers to a quantity that is one larger. Recognize the one more pattern of counting using objects.</p> <p><b>K.CC.B.5:</b> Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.</p>		
Strand	<b>Quantitative Reasoning- Counting and Cardinality</b>		
Standard	<b>QR.C.3</b> Compare numbers.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations	<b>K.CC.C.6:</b> Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another		

	group, e.g., by using matching and counting strategies. Include groups with up to ten objects.  <b>K.CC.C.7:</b> Compare two numbers between 1 and 10 presented as written numerals.		
Strand	<b>Quantitative Reasoning - Numbers and Operations in Base Ten</b>		
Standard	<b>QR.C.4</b> Extend the counting sequence.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations		<b>1.NBT.A.1:</b> Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	
Strand	<b>Quantitative Reasoning - Numbers and Operations in Base Ten</b>		
Standard	<b>QR.C.5</b> Understand place value.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations	<b>K.NBT.A.1:</b> Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., $18 = 10 + 8$ and $10 + 8 = 18$ ); understand that these numbers are composed of	<b>1.NBT.B.2:</b> Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:	<b>2.NBT.A.1:</b> Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: <b>2.NBT.A.1a:</b> 100 can be thought of as a bundle of ten tens - called a "hundred."

	<p>ten ones and one, two, three, four, five, six, seven, eight, or nine ones.</p>	<p><b>1.NBT.B.2a:</b> 10 can be thought of as a bundle of ten ones - called a “ten.”</p> <p><b>1.NBT.B.2b:</b> The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.</p> <p><b>1.NBT.B.2c:</b> The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).</p> <p><b>1.NBT.B.3:</b> Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols <math>&gt;</math>, <math>=</math>, and <math>&lt;</math>.</p>	<p><b>2.NBT.A.1b:</b> The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).</p> <p><b>2.NBT.A.2:</b> Count within 1000; skip-count by 5s, 10s, and 100s. Identify patterns in skip counting at any number. (For example, 37, 47, 57 or 328, 428, 528, etc.)</p> <p><b>2.NBT.A.3:</b> Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.</p> <p><b>2.NBT.A.4:</b> Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record the results of comparisons.</p>
Strand	<b>Quantitative Reasoning - Numbers and Operations in Base Ten</b>		
Standard	<b>QR.C.6</b> Use place value understanding and properties of operations to add and subtract.		
	Childhood		

	Kindergarten	Grade 1	Grade 2
Performance Expectations		<p><b>1.NBT.C.4:</b> Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.</p> <p><b>1.NBT.C.5:</b> Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.</p>	<p><b>2.NBT.B.5:</b> Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</p> <p><b>2.NBT.B.7:</b> Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.</p> <p><b>2.NBT.B.8:</b> Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.</p> <p><b>2.NBT.B.6:</b> Add up to four two-digit numbers using strategies based on place value and properties of operations.</p>



		<p><b>1.NBT.C.6:</b> Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</p>	<p><b>2.NBT.B.9:</b> Explain why addition and subtraction strategies work, using place value and the properties of operations. Explanations may be supported by drawings or objects.</p>
Strand	<b>Quantitative Reasoning - Numbers and Operations in Base Ten</b>		
Standard	<b>QR.C.7</b> Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers and decimals to hundredths.		
	Childhood		
	<p><b>Grade 3</b> A range of algorithms may be used.</p>	<p><b>Grade 4</b> Grade 4 expectations in this <i>strand</i> are limited to whole numbers less than or equal to 1,000,000</p>	<p><b>Grade 5</b></p>
Performance Expectations	<p><b>3.NBT.A.2:</b> Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</p>	<p><b>4.NBT.B.4:</b> Fluently add and subtract multi-digit whole numbers using the standard algorithm.</p>	<p><b>5.NBT.B.5:</b> Fluently multiply multi-digit whole numbers using the standard algorithm.</p>

	<p><b>3.NBT.A.3:</b> Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., <math>9 \times 80</math>, <math>5 \times 60</math>) using strategies based on place value and properties of operations.</p>	<p><b>4.NBT.B.5:</b> Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p> <p><b>4.NBT.B.6:</b> Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>	<p><b>5.NBT.B.6:</b> Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p> <p><b>5.NBT.B.7:</b> Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, money and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</p>
Strand	<b>Quantitative Reasoning - Numbers and Operations in Base Ten</b>		
Standard	<b>QR.C.8</b> Understand the place value system.		
	Childhood		
	Grade 3	Grade 4	Grade 5

	A range of algorithms may be used.	Grade 4 expectations in this <i>strand</i> are limited to whole numbers less than or equal to 1,000,000	
Performance Expectations	<b>3.NBT.A.1:</b> Use place value understanding to round whole numbers to the nearest 10 or 100.	<p><b>4.NBT.A.3:</b> Use place value understanding to round multi-digit whole numbers to any place.</p> <p><b>4.NBT.A.2:</b> Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record the results of comparisons.</p> <p><b>4.NBT.A.1:</b> Recognize that in a multi-digit whole number, a digit in any place represents ten times what it represents in the place to its right. <i>For example, recognize that <math>700 \div 70 = 10</math> by applying concepts of place value and division.</i></p>	<p><b>5.NBT.A.4:</b> Use place value understanding to round decimals to any place.</p> <p><b>5.NBT.A.3:</b> Read, write, and compare decimals to thousandths.</p> <p><b>5.NBT.A.3a:</b> Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., <math>347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)</math>.</p> <p><b>5.NBT.A.3b:</b> Compare two decimals to thousandths based on meanings of the digits in each place, using <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record the results of comparisons.</p> <p><b>5.NBT.A.1:</b> Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.</p> <p><b>5.NBT.A.2:</b> Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied</p>

			or divided by a power of 10. Use whole-number exponents to denote powers of 10.
Strand	<b>Quantitative Reasoning - Numbers and Operations: Fractions</b>		
Standard	<b>QR.C.9</b> Develop and extend the understanding of fractions as numbers, including equivalence and ordering.		
	Childhood		
	Grade 3 Grade 3 expectations in this <i>strand</i> are limited to fractions with denominators 2, 3, 4, 6, and 8.	Grade 4 Grade 4 expectations in this <i>strand</i> are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	Grade 5
Performance Expectations	<p><b>3.NF.A.1:</b> Understand a unit fraction <math>1/b</math> as the quantity formed by 1 part when a whole is partitioned into <math>b</math> equal parts; understand a fraction <math>a/b</math> as the quantity formed by <math>a</math> parts of size <math>1/b</math>.</p> <p><b>3.NF.A.2:</b> Understand a fraction as a number on the number line; represent fractions on a number line diagram.</p> <p><b>3.NF.A.2a:</b> Represent a fraction <math>1/b</math> on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into <math>b</math> equal parts. Recognize that each part has size <math>1/b</math> and that the endpoint of the part based at 0 locates the number <math>1/b</math> on the number line.</p>		

	<p><b>3.NF.A.2b:</b> Represent a fraction <math>a/b</math> on a number line diagram by marking off a lengths <math>1/b</math> from 0. Recognize that the resulting interval has size <math>a/b</math> and that its endpoint locates the number <math>a/b</math> on the number line.</p> <p><b>3.NF.A.3:</b> Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.</p> <p><b>3.NF.A.3a:</b> Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.</p> <p><b>3.NF.A.3b:</b> Recognize and generate simple equivalent fractions, e.g., <math>1/2 = 2/4</math>, <math>4/6 = 2/3</math>. Explain why the fractions are equivalent, e.g., by using a visual fraction model.</p> <p><b>3.NF.A.3c:</b> Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. <i>Examples: Express 3 in the form <math>3 = 3/1</math>; recognize that <math>6/1 = 6</math>; locate <math>4/4</math> and 1 at the same point of a number line diagram.</i></p>	<p><b>4.NF.A.1:</b> Explain why a fraction <math>a/b</math> is equivalent to a fraction <math>(n \times a)/(n \times b)</math> by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions, including fractions greater than 1.</p>	
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	<p><b>3.NF.A.3d:</b> Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols <math>&gt;</math>, <math>=</math>, or <math>&lt;</math>, and justify the conclusions, e.g., by using a visual fraction model.</p>	<p><b>4.NF.A.2:</b> Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as <math>\frac{1}{2}</math>. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols <math>&gt;</math>, <math>=</math>, or <math>&lt;</math>, and justify the conclusions, e.g., by using a visual fraction model.</p>	
Strand	<b>Quantitative Reasoning - Numbers and Operations: Fractions</b>		
Standard	<b>QR.C.10</b> Understand decimal notation for fractions, and compare decimal fractions.		
	Childhood		
	Grade 3	Grade 4	Grade 5
		<p>Grade 4 expectations in this <i>strand</i> are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.</p> <p>Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.</p>	
Performance Expectations		<p><b>4.NF.C.5:</b> Express a fraction with denominator 10 as an equivalent fraction with denominator 100 and use this technique to add two fractions with respective denominators 10 and 100. <i>For</i></p>	

		<p><i>example, express <math>\frac{3}{10}</math> as <math>\frac{30}{100}</math>, and add <math>\frac{3}{10} + \frac{4}{100} = \frac{34}{100}</math>.</i></p> <p><b>4.NF.C.6:</b> Use decimal notation for fractions with denominators 10 or 100. <i>For example, rewrite 0.62 as <math>\frac{62}{100}</math>; describe a length as 0.62 meters; locate 0.62 on a number line diagram.</i></p> <p><b>4.NF.C.7:</b> Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols <math>&gt;</math>, <math>=</math>, or <math>&lt;</math>, and justify the conclusions, e.g., by using a visual model.</p>	
Strand	<b>Quantitative Reasoning - Numbers and Operations: Fractions</b>		
Standard	<b>QR.C.11</b> Use equivalent fractions as a strategy to add and subtract fractions.		
	Childhood		
	Grade 3	Grade 4	Grade 5
		<p>Grade 4 expectations in this <i>strand</i> are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.</p> <p>Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.</p>	

<p>Performance Expectations</p>		<p><b>4.NF.B.3:</b> Understand a fraction <math>a/b</math> with <math>a &gt; 1</math> as a sum of fractions <math>1/b</math>.</p> <p><b>4.NF.B.3a:</b> Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.</p> <p><b>4.NF.B.3b:</b> Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model to build fractions from unit fractions. <i>Examples:</i> <math>3/8 = 1/8 + 1/8 + 1/8</math>; <math>3/8 = 1/8 + 2/8</math>; <math>2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8</math>.</p> <p><b>4.NF.B.3c:</b> Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.</p> <p><b>4.NF.B.3d:</b> Solve word problems involving addition and</p>	<p><b>5.NF.A.1:</b> Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. <i>For example,</i> <math>2/3 + 5/4 = 8/12 + 15/12 = 23/12</math>. <i>(In general,</i> <math>a/b + c/d = (ad + bc)/bd</math>.<i>)</i></p> <p><b>5.NF.A.2:</b> Solve word problems involving addition and subtraction of fractions</p>
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		subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. <i>For example, recognize an incorrect result <math>2/5 + 1/2 = 3/7</math>, by observing that <math>3/7 &lt; 1/2</math>.</i>
Strand	<b>Quantitative Reasoning - Numbers and Operations: Fractions</b>		
Standard	<b>QR.C.12</b> Apply and extend previous understandings of multiplication and division to multiply and divide fractions.		
	Childhood		
	Grade 3	Grade 4	Grade 5 Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.
Performance Expectations		<p><b>4.NF.B.4:</b> Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.</p> <p><b>4.NF.B.4a:</b> Understand a fraction <math>a/b</math> as a multiple of <math>1/b</math>. <i>For example, use a visual fraction model to represent <math>5/4</math> as the product <math>5 \times (1/4)</math>, recording the conclusion by the equation <math>5/4 = 5 \times (1/4)</math>.</i></p> <p><b>4.NF.B.4b:</b> Understand a multiple of <math>a/b</math> as a multiple of <math>1/b</math> and use this understanding to multiply a</p>	<p><b>5.NF.B.4:</b> Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.</p> <p><b>5.NF.B.4a:</b> Interpret the product <math>(a/b) \times q</math> as a parts of a partition of <math>q</math> into <math>b</math> equal parts; equivalently, as the result of a sequence of operations <math>a \times q \div b</math>. <i>For example, use a visual fraction model to show <math>(2/3) \times 4 = 8/3</math>, and create a story context for this equation. Do the same with <math>(2/3) \times (4/5) = 8/15</math>. (In general, <math>(a/b) \times (c/d) = (ac)/(bd)</math>.</i></p>

		<p>fraction by a whole number. <i>For example, use a visual fraction model to express <math>3 \times (2/5)</math> as <math>6 \times (1/5)</math>, recognizing this product as <math>6/5</math>. (In general, <math>n \times (a/b) = (n \times a)/b</math>.)</i></p> <p><b>4.NF.B.4c:</b> Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. <i>For example, if each person at a party will eat <math>3/8</math> of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?</i></p>	<p><b>5.NF.B.4b:</b> Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas.</p> <p><b>5.NF.B.3:</b> Interpret a fraction as division of the numerator by the denominator (<math>a/b = a \div b</math>). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. <i>For example, interpret <math>3/4</math> as the result of dividing 3 by 4, noting that <math>3/4</math> multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size <math>3/4</math>. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?</i></p> <p><b>5.NF.B.5:</b> Interpret multiplication scaling (resizing), by:</p> <p><b>5.NF.B.5a:</b> Comparing the size of a product to the size of one factor on</p>
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			<p>the basis of the size of the other factor, without performing the indicated multiplication.</p> <p><b>5.NF.B.5b:</b> Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence <math>a/b = (n \times a)/(n \times b)</math> to the effect of multiplying <math>a/b</math> by 1.</p> <p><b>5.NF.B.6:</b> Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</p> <p><b>5.NF.B.7:</b> Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.</p> <p><b>5.NF.B.7a:</b> Interpret division of a unit fraction by a non-zero whole number and compute such quotients. <i>For example, create a story context for <math>(1/3) \div 4</math>, and use a visual fraction model to show the quotient. Use the</i></p>
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			<p><i>relationship between multiplication and division to explain that <math>(1/3) \div 4 = 1/12</math> because <math>(1/12) \times 4 = 1/3</math>.</i></p> <p><b>5.NF.B.7b:</b> Interpret division of a whole number by a unit fraction and compute such quotients. <i>For example, create a story context for <math>4 \div (1/5)</math>, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that <math>4 \div (1/5) = 20</math> because <math>20 \times (1/5) = 4</math>.</i></p> <p><b>5.NF.B.7c:</b> Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. <i>For example, how much chocolate will each person get if 3 people share <math>1/2</math> lb of chocolate equally? How many <math>1/3</math>-cup servings are in 2 cups of raisins?</i></p>
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Strand	<b>Quantitative Reasoning- Ratio and Proportional Relationships</b>
Standard	<b>QR.EA.1</b> Understand ratio and rate concepts and use ratio and rate reasoning to solve problems.
	Early Adolescence
	Grades 6-8

Performance Expectations	<p><b>6.RP.A.1:</b> Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. <i>For example, "The ratio of wings to beaks of the chickadees in the pine tree was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."</i></p> <p><b>6.RP.A.2:</b> Understand the concept of a unit rate <math>a/b</math> associated with a ratio <math>a:b</math> with <math>b \neq 0</math> and use rate language in the context of a ratio relationship. <i>For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is <math>3/4</math> cup of flour for each cup of sugar." "We paid \$75 for 5 lobsters, which is a rate of \$15 per lobster." Expectations for unit rates in this grade are limited to non-complex fractions.</i></p> <p><b>6.RP.A.3:</b> Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</p> <p><b>6.RP.A.3a:</b> Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</p> <p><b>6.RP.A.3b:</b> Solve unit rate problems including those involving unit pricing and constant speed. <i>For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?</i></p> <p><b>6.RP.A.3c:</b> Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.</p> <p><b>6.RP.A.3d:</b> Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</p>
Strand	<b>Quantitative Reasoning- Ratio and Proportional Relationships</b>
Standard	<b>QR.EA.2</b> Analyze proportional relationships and use them to solve real-world and mathematical problems.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p><b>7.RP.A.1:</b> Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. <i>For example, if a person walks <math>1/2</math> mile in each <math>1/4</math> hour, compute the unit rate as the complex fraction <math>\frac{1/2}{1/4}</math> miles per hour, equivalently 2 miles per hour.</i></p>

	<p><b>7.RP.A.2:</b> Recognize and represent proportional relationships between quantities.</p> <p><b>7.RP.A.2a:</b> Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.</p> <p><b>7.RP.A.2b:</b> Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.</p> <p><b>7.RP.A.2c:</b> Represent proportional relationships by equations. <i>For example, if the total cost <math>t</math> is proportional to the number <math>n</math> of items purchased at a constant price <math>p</math>, the relationship between the total cost and the number of items can be expressed as <math>t = pn</math>.</i></p> <p><b>7.RP.A.2d:</b> Explain what a point <math>(x, y)</math> on the graph of a proportional relationship means in terms of the situation, with special attention to the points <math>(0, 0)</math> and <math>(1, r)</math> where <math>r</math> is the unit rate.</p> <p><b>7.RP.A.3:</b> Use proportional relationships to solve multistep ratio, rate, and percent problems. <i>Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</i></p>
Strand	<b>Quantitative Reasoning- The Number System</b>
Standard	<b>QR.EA.3</b> Apply and extend previous understandings of operations with whole numbers to rational numbers.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p><b>6.NS.A.1:</b> Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and/or equations to represent the problem. <i>For example, create a story context for <math>(2/3) \div (3/4)</math> and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that <math>(2/3) \div (3/4) = 8/9</math> because <math>3/4</math> of <math>8/9</math> is <math>2/3</math>. (In general, <math>(a/b) \div (c/d) = ad/bc</math>.)</i></p> <p><b>6.NS.B.3:</b> Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.</p> <p><b>7.NS.A.1:</b> Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.</p>

	<p><b>7.NS.A.1a:</b> Describe situations in which opposite quantities combine to make 0. <i>For example, a hydrogen atom has a zero charge because its two constituents are oppositely charged.</i></p> <p><b>7.NS.A.1b:</b> Understand <math>p + q</math> as the number located a distance <math> q </math> from <math>p</math>, in the positive or negative direction depending on whether <math>q</math> is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.</p> <p><b>7.NS.A.1c:</b> Understand subtraction of rational numbers as adding the additive inverse, <math>p - q = p + (-q)</math>. Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts.</p> <p><b>7.NS.A.1d:</b> Apply properties of operations as strategies to add and subtract rational numbers.</p> <p><b>7.NS.A.2:</b> Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.</p> <p><b>7.NS.A.2a:</b> Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as <math>(-1)(-1) = 1</math> and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.</p> <p><b>7.NS.A.2b:</b> Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If <math>p</math> and <math>q</math> are integers, then <math>-(p/q) = (-p)/q = p/(-q)</math>. Interpret quotients of rational numbers by describing real-world contexts.</p> <p><b>7.NS.A.2c:</b> Apply properties of operations as strategies to multiply and divide rational numbers.</p> <p><b>7.NS.A.2d:</b> Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.</p> <p><b>7.NS.A.3:</b> Solve real-world and mathematical problems involving the four operations with rational numbers. <i>Computations with rational numbers extend the rules for manipulating fractions to complex fractions.</i></p>
Strand	<b>Quantitative Reasoning- The Number System</b>
Standard	<b>QR.EA.4</b> Compute fluently with multi-digit whole numbers and find common factors and multiples.
	Early Adolescence
	Grades 6-8
Performance Expectations	<b>6.NS.B.2:</b> Fluently divide multi-digit numbers using the standard algorithm.

	<b>6.NS.B.4:</b> Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. ( <i>For example: Use prime factorization to find the greatest common factor</i> ); Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two relatively prime numbers. <i>For example, express <math>36 + 8</math> as <math>4(9 + 2)</math>.</i>
Strand	<b>Quantitative Reasoning - The Number System</b>
Standard	<b>QR.EA.5</b> Apply and extend previous understandings of numbers to the system of rational numbers.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p><b>6.NS.C.5:</b> Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative rational numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.</p> <p><b>6.NS.C.6:</b> Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.</p> <p><b>6.NS.C.6a:</b> Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., <math>-(-3) = 3</math>, and that 0 is its own opposite.</p> <p><b>6.NS.C.6b:</b> Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.</p> <p><b>6.NS.C.6c:</b> Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.</p> <p><b>6.NS.C.7:</b> Understand ordering and absolute value of rational numbers.</p> <p><b>6.NS.C.7a:</b> Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. <i>For example, interpret <math>-3 &gt; -7</math> as a statement that -3 is located to the right of -7 on a number line oriented from left to right.</i></p> <p><b>6.NS.C.7b:</b> Write, interpret, and explain statements of order for rational numbers in real-world contexts. <i>For example, write <math>-3^{\circ}C &gt; -7^{\circ}C</math> to express the fact that <math>-3^{\circ}C</math> is warmer than <math>-7^{\circ}C</math>.</i></p>



	<p><b>6.NS.C.7c:</b> Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. <i>For example, for an account balance of -30 dollars, write <math> -30  = 30</math> to describe the size of the debt in dollars.</i></p> <p><b>6.NS.C.7d:</b> Distinguish comparisons of absolute value from statements about order. <i>For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.</i></p> <p><b>6.NS.C.8:</b> Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</p>
Strand	<b>Quantitative Reasoning- The Number System</b>
Standard	<b>QR.EA.6</b> Know that there are numbers that are not rational, and approximate them by rational numbers.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p><b>8.NS.A.1:</b> Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansions terminate in 0s or eventually repeats and convert a decimal expansion into a rational number.</p> <p><b>8.NS.A.2:</b> Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., <math>\pi^2</math>). <i>For example, by truncating the decimal expansion of <math>\sqrt{2}</math>, show that <math>\sqrt{2}</math> is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.</i></p>

Strand	<b>Quantitative Reasoning - Number and Quantity: The Real Number System</b>
Standard	<b>QR.A.1</b> Extend the properties of exponents to rational exponents.
	Adolescence
	Grades 9-Diploma
Performance Expectations	<b>HSN.RN.A.1:</b> Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational

	<p>exponents. For example, we define <math>5^{1/3}</math> to be the cube root of 5 because we want <math>(5^{1/3})^3 = 5^{(1/3)3}</math> to hold, so <math>(5^{1/3})^3</math> must equal 5.</p> <p><b>HSN.RN.A.2:</b> Rewrite expressions involving radicals and rational exponents using the properties of exponents.</p>
Strand	<b>Quantitative Reasoning- Number and Quantity: The Real Number System</b>
Standard	<b>QR.A.2</b> Use properties of rational and irrational numbers.
	Adolescence
	Grades 9-Diploma
Performance Expectations	<b>HSN.RN.B.3:</b> Explain when and why the sum or product of two rational and/or irrational numbers is rational or irrational.
Strand	<p><b>Quantitative Reasoning - Number and Quantity: Quantities</b></p> <p><b>Modeling Standards:</b> Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).</p>
Standard	<b>QR.A.3</b> Reason quantitatively and use units to solve problems. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	<p><b>HSN.Q.A.1:</b> Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. Example: Marlena made a scale drawing of the sand volleyball court at her summer camp. The drawing of the volleyball court is 6 cm long by 3 cm wide. The actual volleyball court is 18 meters long. What scale did Marlena use for the drawing? ★</p> <p><b>HSN.Q.A.2:</b> Define appropriate quantities for the purpose of descriptive modeling. Example: If a town in Aroostook county with a population of 1254 people is projected to double in size every 105 years, what will the population be 315 years from now? ★</p>

	<p><b>HSN.Q.A.3:</b> Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. Example: The label on a <math>\frac{1}{2}</math> - liter bottle of flavored water bottled in Maine indicates that one serving of 8 ounce contains 60 calories. The label also says that the full bottle contains 130 calories. Is this the actual amount or the estimated amount of calories in this bottle? How would you explain any discrepancy? ★</p>
Strand	<p><b>Quantitative Reasoning - Number and Quantity: Complex Number System</b></p> <p>The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.</p>
Standard	<p><b>QR.A.4 (+)</b> Perform arithmetic operations with complex numbers.</p>
	<p>Adolescence</p>
	<p>Grades 9-Diploma</p>
Performance Expectations	<p><b>(+) HSN.CN.A.1:</b> Know there is a complex number <math>i</math> (which is a non-real number) such that <math>i^2 = -1</math>, and every complex number has the form <math>a + bi</math> with <math>a</math> and <math>b</math> real.</p> <p><b>(+) HSN.CN.A.2:</b> Use the relation <math>i^2 = -1</math> and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</p> <p><b>(+) HSN.CN.A.3:</b> Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.</p>
Strand	<p><b>Quantitative Reasoning - Number and Quantity: Complex Number System</b></p> <p>The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.</p>
Standard	<p><b>QR.A.5 (+)</b> Represent complex numbers and their operations on the complex plane.</p>
	<p>Adolescence</p>
	<p>Grades 9-Diploma</p>
Performance Expectations	<p><b>(+) HSN.CN.B.4:</b> Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.</p>

	<p><b>(+) HSN.CN.B.5:</b> Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. <i>For example, <math>(-1 + \sqrt{3}i)^3 = 8</math> because <math>(-1 + \sqrt{3}i)</math> has modulus 2 and argument <math>120^\circ</math>.</i></p> <p><b>(+) HSN.CN.B.6:</b> Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.</p>
Strand	<p><b>Quantitative Reasoning - Number and Quantity: Complex Number System</b></p> <p>The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.</p>
Standard	<b>QR.A.6 (+)</b> Use complex numbers in polynomial identities and equations.
	Adolescence
	Grades 9-Diploma
Performance Expectations	<p><b>(+) HSN.CN.C.7:</b> Solve quadratic equations with real coefficients that have complex solutions.</p> <p><b>(+) HSN.CN.C.8:</b> Extend polynomial identities to the complex numbers. <i>For example, rewrite <math>x^2 + 4</math> as <math>(x + 2i)(x - 2i)</math>.</i></p> <p><b>(+) HSN.CN.C.9:</b> Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.</p>
Strand	<p><b>Quantitative Reasoning - Number and Quantity: Vector and Matrix Quantities</b></p> <p>The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.</p>
Standard	<b>QR.A.7 (+)</b> Represent and model with vector quantities.
	Adolescence
	Grades 9-Diploma
Performance Expectations	<b>(+) HSN.VM.A.1:</b> Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{v}$ , $ \mathbf{v} $ , $\ \mathbf{v}\ $ , $v$ ).

	<p><b>(+) HSN.VM.A.2:</b> Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.</p> <p><b>(+) HSN.VM.A.3:</b> Solve problems involving velocity and other quantities that can be represented by vectors.</p>
Strand	<p><b>Quantitative Reasoning - Number and Quantity: Vector and Matrix Quantities</b>  The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.</p>
Standard	<b>QR.A.8 (+)</b> Perform operations on vectors.
	Adolescence
	Grades 9-Diploma
Performance Expectations	<p><b>(+) HSN.VM.B.4:</b> Add and subtract vectors.</p> <p><b>(+) HSN.VM.B4a:</b> Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.</p> <p><b>(+) HSN.VM.B4b:</b> Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.</p> <p><b>(+) HSN.VM.B4c:</b> Understand vector subtraction <math>\mathbf{v} - \mathbf{w}</math> as <math>\mathbf{v} + (-\mathbf{w})</math>, where <math>-\mathbf{w}</math> is the additive inverse of <math>\mathbf{w}</math>, with the same magnitude as <math>\mathbf{w}</math> and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.</p> <p><b>(+) HSN.VM.B.5:</b> Multiply a vector by a scalar.</p> <p><b>(+) HSN.VM.B5a:</b> Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as <math>c(v_x, v_y) = (cv_x, cv_y)</math>.</p> <p><b>(+) HSN.VM.B5b:</b> Compute the magnitude of a scalar multiple <math>c\mathbf{v}</math> using <math>\ c\mathbf{v}\  =  c \mathbf{v}</math>. Compute the direction of <math>c\mathbf{v}</math> knowing that when <math> c \mathbf{v} \neq 0</math>, the direction of <math>c\mathbf{v}</math> is either along <math>\mathbf{v}</math> (for <math>c &gt; 0</math>) or against <math>\mathbf{v}</math> (for <math>c &lt; 0</math>).</p>
Strand	<p><b>Quantitative Reasoning - Number and Quantity: Vector and Matrix Quantities</b>  The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.</p>
Standard	<b>QR.A.9 (+)</b> Perform operations on matrices and use matrices in applications.
	Adolescence

Grades 9-Diploma

	Grades 9-Diploma
Performance Expectations	<p><b>(+) HSN.VM.C.6:</b> Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.</p> <p><b>(+) HSN.VM.C.7:</b> Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.</p> <p><b>(+) HSN.VM.C.8:</b> Add, subtract, and multiply matrices of appropriate dimensions.</p> <p><b>(+) HSN.VM.C.9:</b> Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.</p> <p><b>(+) HSN.VM.C.10:</b> Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.</p> <p><b>(+) HSN.VM.C.11:</b> Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.</p> <p><b>(+) HSN.VM.C.12:</b> Work with <math>2 \times 2</math> matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.</p>

**Definitions:**

Strand: A body of knowledge in a content area identified by a simple title.

Standard: Enduring understandings and skills that students can apply and transfer to contexts that are new to the student.

Performance Expectation: Building blocks to the standard and measurable articulations of what the student understands and can do.