# Mathematics Standards Introduction

A strong mathematics education depends upon a clear understanding of its interrelated concepts, skills and practices to ensure students are on the pathway to success in their academic careers. The knowledge and skills students need to be prepared for mathematics in college, career, and life are woven throughout the K-12 mathematics performance expectations.

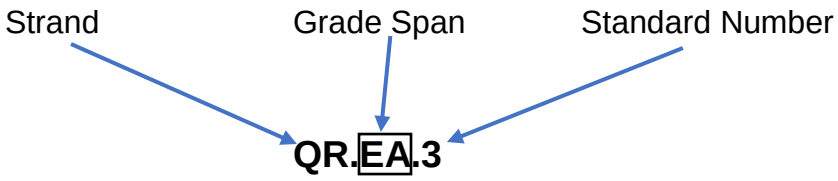
# Outline of Mathematics Strands and Standards

These mathematical performance expectations are building blocks to standards. The standards are grouped into four strands:

- **Quantitative Reasoning** (Blue)**:** Counting and Cardinality, Number and Operations in Base Ten, Number and Operations Fractions, Ratio and Proportional Relations, The Number System, and Number and Quantity.
- **Algebraic Reasoning** (Green)**:** Operations and Algebraic Thinking, Expressions and Equations, Functions, and Algebra
- **Geometric Reasoning** (Red)**:** Geometry
- **Statistical Reasoning** (Purple)**:** Measurement and Data, Statistics and Probability

These mathematical performance expectations are broken into three grade spans: **C**hildhood (K-5), **E**arly **A**dolescence (6-8), and **A**dolescence (9-Diploma). The strands are color-coded, as indicated above, for continuity throughout the grade spans. Standards do not work in isolation, they are connected through and across strands.

# How to Read the Standards




Within the high school performance expectations, modeling is woven throughout the four strands and is denoted with a star (★). The high school standards also contain some performance expectations which are denoted by a plus (**+**). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.

## The Guiding Principles & Standards for Mathematical Practice

The Guiding Principles influence education in Maine and should be reflected throughout Mathematics curriculum. The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Full descriptions of the Guiding Principles and Standards for Mathematical Practice can be found in the Supplemental Material. Examples of how students can show evidence of those **Guiding Principles** and **Standards for Mathematical Practice** **may** include:

## Guiding Principles

**A.** **A clear and effective communicator:** Students will use written, oral, symbolic, and visual forms of expression to communicate mathematically.

**B.** **A self-directed and lifelong learner:** Students generate and persevere in solving questions while demonstrating a growth mindset.

**C.** **A creative and practical problem solver:** Students will pose and solve mathematical problems by using a variety of strategies that connect to real-world examples.

**D.** **A responsible and involved citizen:** Students make sense of the world around them through mathematics including economic literacy.

**E.** **An integrative and informed thinker:** Students connect mathematics to other learning by understanding the interrelationships of mathematical ideas and the role math plays in other disciplines and life.

## Standards for Mathematical Practice

**1.** **Make sense of problems and persevere in solving them:** Students will plan strategies to use and persevere in solving math problems.

**2.** **Reason abstractly and quantitatively:** Students will think about numbers in many ways and make sense of numerical relationships as they solve problems.

**3.** **Construct viable arguments and critique the reasoning of others:** Students will explain their thinking and make sense of the thinking of others.

**4.** **Model with mathematics:** Students will use representations to show their thinking in a variety of ways.

**5.** **Use appropriate tools strategically:** Students will use math tools such as tables, diagrams, and technology to explore and deepen their understanding of concepts.

**6.** **Attend to precision:** Students will use precise mathematical language and check their work for accuracy.

**7.** **Look for and make use of structure:** Students will use their current mathematical understandings to identify patterns and structure to make sense of new learning.

**8.** **Look for and express regularity in repeated reasoning:** Students will look for patterns and rules to help create general methods and shortcuts that can be applied to similar mathematical problems.

## Geometric Reasoning

Geometric reasoning is the use of critical thinking, logical argument and spatial reasoning to solve problems and find new relationships. Students must first have a critical understanding of any underlying assumptions and relationships. This allows them to develop coherent knowledge and apply their reasoning skills. In this K-5 strand, students will develop an understanding of the attributes of two- and three-dimensional shapes and apply this knowledge to real-world problems. Students will also be introduced to the coordinate system.

Students in grades 6-8 work with two- and three-dimensional objects to reason about relationships among shapes. They learn to calculate area, surface area, volume, and circumference using multiple methods including decomposing shapes so that they can develop, justify, and use formulas including the Pythagorean Theorem and its converse. They use scale drawings and informal constructions to gain familiarity with the relationships between angles formed by intersecting lines and transformations.

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more complex definitions and reasoning of proofs. Students make geometric constructions using a variety of technological tools and connect these explorations to reasoning and proofs. Attributes of parallel lines intersected by a transversal are further developed and extended into properties of triangles, quadrilaterals, and regular polygons as well as circles using informal and formal reasoning. Fundamental to the concepts of congruence, similarity, and symmetry are transformations which can preserve distance and angles.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity. The Pythagorean Theorem along with these ratios are fundamental in many real-world and theoretical situations. Correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. Concepts of two- and three-dimensional shapes are explored using algebraic formulas and modeling. Students are encouraged to extend their geometric reasoning through the exploration of trigonometric identities and properties of conic sections.

| Strand | **Geometric Reasoning - Geometry** | | |
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| Standard | **GR.C.1** Identify, describe, analyze, compare, create, and compose shapes based on their attributes. | | |
| | Childhood | | |
| | Kindergarten | Grade 1<br>Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term. | Grade 2<br>Sizes are compared directly or visually, not compared by measuring. |
| Performance Expectations | **K.G.A.1:** Describe objects in the environment using names of shapes and describe the relative positions of these objects using terms such as *above*, *below*, *beside, in front of*, *behind*, and *next to*.<br><br>**K.G.A.2:** Correctly name shapes regardless of their orientations or overall size.<br><br>**K.G.A.3:** Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").<br><br>**K.G.B.4:** Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal | **1.G.A.1:** Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes. | **2.G.A.1:** Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals (including squares, rectangles, rhombuses, and trapezoids) pentagons, hexagons, and cubes. Sizes are compared directly or visually, not compared by measuring. |

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| | language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).<br><br>**K.G.B.5:** Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.<br><br>**K.G.B.6:** Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle*?" | **1.G.A.2:** Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.<br><br>**1.G.A.3:** Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. | **2.G.A.2:** Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.<br><br>**2.G.A.3:** Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two |

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| | | Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. | halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. |
| Strand | **Geometric Reasoning - Geometry** | | |
| Standard | **GR.C.2** Analyze, compare, create, and compose shapes based on their attributes. | | |
| | Childhood | | |
| | Grade 3 | Grade 4 | Grade 5 |
| Performance Expectations | **3.G.A.1:** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.<br><br>**3.G.A.2:** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each* | | **5.G.B.3:** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. |

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| | *part as 1/4 of the area of the shape*. | | **5.G.B.4:** Classify two-dimensional figures in a hierarchy based on properties. (e.g., all rectangles are parallelograms, because they are all quadrilaterals with two pairs of opposite sides parallel.) |
| Strand | **Geometric Reasoning - Geometry** | | |
| Standard | **GR.C.3** Draw and identify lines and angles and classify shapes by properties of their lines and angles. | | |
| | Childhood | | |
| | Grade 3 | Grade 4 | Grade 5 |
| Performance Expectations | | **4.G.A.1:** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.<br><br>**4.G.A.2:** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category and identify right triangles.<br><br>**4.G.A.3:** Recognize a line of symmetry for a two-dimensional | |

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| | | figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. | |
| Strand | **Geometric Reasoning - Geometry** | | |
| Standard | **GR.C.4** Graph points on the coordinate plane to solve real-world and mathematical problems. | | |
| | Childhood | | |
| | Grade 3 | Grade 4 | Grade 5 |
| Performance Expectations | | | **5.G.A.1:** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). |

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| | | | **5.G.A.2:** Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation. |

| Strand | **Geometric Reasoning - Geometry** |
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| Standard | **GR.EA.1** Solve real-world and mathematical problems involving angle measure, area, surface area, and volume. |
| | Early Adolescence |
| | Grades 6-8 |
| Performance Expectations | **6.G.A.1:** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.<br><br>**6.G.A.2:** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = B h (where B stands for the area of the base) to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.<br><br>**6.G.A.3:** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.<br><br>**6.G.A.4:** Represent three-dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. |

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| | **7.G.B.4:** Know that a circle is a two-dimensional shape created by connecting all the points equidistant from a fixed point called the center of the circle. Understand and describe the relationships among the radius, diameter, circumference and area of a circle. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.<br><br>**7.G.B.5:** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.<br><br>**7.G.B.6:** Solve real-world and mathematical problems involving area, volume and surface area of two- and/or three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.<br><br>**8.G.C.9:** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. |
| Strand | **Geometric Reasoning - Geometry** |
| Standard | **GR.EA.2** Draw, construct, and describe geometrical figures and describe the relationships between them. |
| | Early Adolescence |
| | Grades 6-8 |
| Performance Expectations | **7.G.A.1:** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.<br><br>**7.G.A.2:** Draw (freehand, with ruler and protractor, and with technology) two-dimensional geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.<br><br>**7.G.A.3:** Describe the shape of the cross-section two-dimensional face of the figures that results from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. |
| Strand | **Geometric Reasoning - Geometry** |
| Standard | **GR.EA.3** Understand congruence and similarity using physical models, transparencies, or geometry software, |
| | Early Adolescence |
| | Grades 6-8 |

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| Performance Expectations | **8.G.A.1:** Verify experimentally the properties of rotations, reflections, and translations:<br>**8.G.A.1a:** Lines are taken to lines, and line segments to line segments of the same length.<br>**8.G.A.1b:** Angles are taken to angles of the same measure.<br>**8.G.A.1c:** Parallel lines are taken to parallel lines.<br><br>**8.G.A.2:** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.<br><br>**8.G.A.3:** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.<br><br>**8.G.A.4:** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.<br><br>**8.G.A.5:** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so*. |
| Strand | **Geometric Reasoning - Geometry** |
| Standard | **GR.EA.4** Understand and apply the Pythagorean Theorem. |
| | Early Adolescence |
| | Grades 6-8 |
| Performance Expectations | **8.G.B.6:** Explain a proof of the Pythagorean Theorem and its converse using pictures, diagrams, narratives or models.<br><br>**8.G.B.7:** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.<br><br>**8.G.B.8:** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. |

| Strand | **Geometric Reasoning - Geometry: Congruence** |
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| Standard | **GR.A.1** Experiment with transformations in the plane. |
| | Adolescence |
| | Grades 9-Diploma |
| Performance Expectations | **HSG.CO.A.1:** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.<br><br>**HSG.CO.A.2:** Represent transformations in the plane using, e.g., transparencies and/or geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).<br><br>**HSG.CO.A.3:** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.<br><br>**HSG.CO.A.4:** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.<br><br>**HSG.CO.A.5:** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. |
| Strand | **Geometric Reasoning - Geometry: Congruence** |
| Standard | **GR.A.2** Understand congruence in terms of rigid motions. |
| | Adolescence |
| | Grades 9-Diploma |
| Performance Expectations | **HSG.CO.B.6:** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.<br><br>**HSG.CO.B.7:** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. |

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| | **HSG.CO.B.8:** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. |
| Strand | **Geometric Reasoning - Geometry: Congruence** |
| Standard | **GR.A.3** Prove geometric theorems and when appropriate, the converse of theorems. |
| | Adolescence |
| | Grades 9-Diploma |
| Performance Expectations | **HSG.CO.C.9:** Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints*.<br><br>**HSG.CO.C.10:** Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point*.<br><br>**HSG.CO.C.11:** Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals*. |
| Strand | **Geometric Reasoning - Geometry: Congruence** |
| Standard | **GR.A.4** Make geometric constructions. |
| | Adolescence |
| | Grades 9-Diploma |
| Performance Expectations | **HSG.CO.D.12:** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line*.<br><br>**HSG.CO.D.13:** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. |
| Strand | **Geometric Reasoning - Geometry: Similarity, Right Triangles, & Trigonometry** |

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| Standard | **GR.A.5** Understand similarity in terms of similarity transformations. |
| | Adolescence |
| | Grades 9-Diploma |
| Performance Expectations | **HSG.SRT.A.1:** Verify experimentally the properties of dilations given by a center and a scale factor:<br>**HSG.SRT.A.1a:** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.<br>**HSG.SRT.A.1b:** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.<br><br>**HSG.SRT.A.2:** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.<br><br>**HSG.SRT.A.3:** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. |
| Strand | **Geometric Reasoning - Geometry: Similarity, Right Triangles, & Trigonometry** |
| Standard | **GR.A.6** Prove theorems involving similarity using a variety of ways of writing proofs, showing validity of underlying reasoning. |
| | Adolescence |
| | Grades 9-Diploma |
| Performance Expectations | **HSG.SRT.B.4:** Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*<br><br>**HSG.SRT.B.5:** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. |
| Strand | **Geometric Reasoning - Geometry: Similarity, Right Triangles, & Trigonometry**<br><br>**Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).** |
| Standard | **GR.A.7** Define trigonometric ratios and solve problems involving right triangles. |
| | Adolescence |
| | Grades 9-Diploma |

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| Performance Expectations | **HSG.SRT.C.6:** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.<br><br>**HSG.SRT.C.7:** Explain and use the relationship between the sine and cosine of complementary angles.<br><br>**HSG.SRT.C.8:** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. For example, find the current height of the tallest pine tree in Maine using the angle of elevation and the distance from the tree. ★ |
| Strand | **Geometric Reasoning - Geometry: Similarity, Right Triangles, & Trigonometry**<br><br>**The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.** |
| Standard | **GR.A.8 (+)** Apply trigonometry to general triangles. |
| | Adolescence |
| | Grades 9-Diploma |
| Performance Expectations | **(+) HSG.SRT.D.9:** Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.<br><br>**(+) HSG.SRT.D.10:** Prove the Laws of Sines and Cosines and use them to solve problems.<br><br>**(+) HSG.SRT.D.11:** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). |
| Strand | **Geometric Reasoning - Geometry: Circle**<br><br>**The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.** |
| Standard | **GR.A.9** Understand and apply theorems about circles. |
| | Adolescence |
| | Grades 9-Diploma |
| Performance Expectations | **HSG.C.A.1:** Prove that all circles are similar. |

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| | **HSG.C.A.2:** Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*<br><br>**HSG.C.A.3:** Construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.<br><br>**(+) HSG.C.A.4:** Construct a tangent line from a point outside a given circle to the circle. |
| Strand | **Geometric Reasoning - Geometry: Circle** |
| Standard | **GR.A.10** Find arc lengths and areas of sectors of circles. |
| | Adolescence |
| | Grades 9-Diploma |
| Performance Expectations | **HSG.C.B.5:** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. |
| Strand | **Geometric Reasoning - Geometry: Expressing Geometric Properties with Equations**<br><br>**The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.** |
| Standard | **GR.A.11** Translate between the geometric description and the equation for a conic section. |
| | Adolescence |
| | Grades 9-Diploma |
| Performance Expectations | **HSG.GPE.A.1:** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.<br><br>**HSG.GPE.A.2:** Derive the equation of a parabola given a focus and directrix.<br><br>**(+) HSG.GPE.A.3:** Derive the equations of ellipses and hyperbolas given the foci ~~and directrix~~, using the fact that the sum or difference of distances from the foci is constant. |

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| Strand | **Geometric Reasoning - Geometry: Expressing Geometric Properties with Equations**<br><br>**Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).** |
| Standard | **GR.A.12** Use coordinates to prove simple geometric theorems algebraically. |
| | Adolescence |
| | Grades 9-Diploma |
| Performance Expectations | **HSG.GPE.B.4:** Use coordinates to prove simple geometric theorems algebraically including the distance formula and its relationship to the Pythagorean Theorem. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*<br><br>**HSG.GPE.B.5:** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).<br><br>**HSG.GPE.B.6:** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.<br><br>**HSG.GPE.B.7:** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ★ |
| Strand | **Geometric Reasoning - Geometry: Geometric Measurements & Dimension**<br><br>**Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).**<br><br>**The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.** |
| Standard | **GR.A.13** Explain volume formulas and use them to solve problems. |
| | Adolescence |
| | Grades 9-Diploma |

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| Performance Expectations | **HSG.GMD.A.1:** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and/or informal limit arguments*.<br><br>**(+) HSG.GMD.A.2:** Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.<br><br>**HSG.GMD.A.3:** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★ |
| Strand | **Geometric Reasoning - Geometry: Geometric Measurements & Dimension** |
| Standard | **GR.A.14** Visualize relationships between two-dimensional and three-dimensional objects. |
| | Adolescence |
| | Grades 9-Diploma |
| Performance Expectations | **HSG.GMD.B.4:** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. |
| Strand | **Geometric Reasoning - Geometry: Modeling with Geometry**<br><br>**Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).** |
| Standard | **GR.A.15** Apply geometric concepts in modeling situations. ★ |
| | Adolescence |
| | Grades 9-Diploma |
| Performance Expectations | **HSG.MG.A.1:** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★<br><br>**HSG.MG.A.2:** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). ★<br><br>**HSG.MG.A.3:** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★ |

**Definitions:**

Strand: A body of knowledge in a content area identified by a simple title.

Standard: Enduring understandings and skills that students can apply and transfer to contexts that are new to the student.

Performance Expectation: Building blocks to the standard and measurable articulations of what the student understands and can do.