

Mathematics Standards Introduction

A strong mathematics education depends upon a clear understanding of its interrelated concepts, skills and practices to ensure students are on the pathway to success in their academic careers. The knowledge and skills students need to be prepared for mathematics in college, career, and life are woven throughout the K-12 mathematics performance expectations.

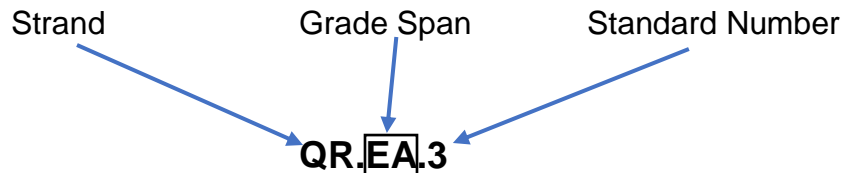
Outline of Mathematics Strands and Standards

These mathematical performance expectations are building blocks to standards. The standards are grouped into four strands:

- **Quantitative Reasoning** (Blue): Counting and Cardinality, Number and Operations in Base Ten, Number and Operations Fractions, Ratio and Proportional Relations, The Number System, and Number and Quantity.
- **Algebraic Reasoning** (Green): Operations and Algebraic Thinking, Expressions and Equations, Functions, and Algebra
- **Geometric Reasoning** (Red): Geometry
- **Statistical Reasoning** (Purple): Measurement and Data, Statistics and Probability

These mathematical performance expectations are broken into three grade spans: **Childhood** (K-5), **Early Adolescence** (6-8), and **Adolescence** (9-Diploma). The strands are color-coded, as indicated above, for continuity throughout the grade spans. Standards do not work in isolation, they are connected through and across strands.

How to Read the Standards



Within the high school performance expectations, modeling is woven throughout the four strands and is denoted with a star (★). The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.

The Guiding Principles & Standards for Mathematical Practice

The Guiding Principles influence education in Maine and should be reflected throughout Mathematics curriculum. The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Full descriptions of the Guiding Principles and Standards for Mathematical Practice can be found in the Supplemental Material. Examples of how students can show evidence of those **Guiding Principles** and **Standards for Mathematical Practice** may include:

Guiding Principles

- A. **A clear and effective communicator:** Students will use written, oral, symbolic, and visual forms of expression to communicate mathematically.
- B. **A self-directed and lifelong learner:** Students generate and persevere in solving questions while demonstrating a growth mindset.
- C. **A creative and practical problem solver:** Students will pose and solve mathematical problems by using a variety of strategies that connect to real-world examples.
- D. **A responsible and involved citizen:** Students make sense of the world around them through mathematics including economic literacy.
- E. **An integrative and informed thinker:** Students connect mathematics to other learning by understanding the interrelationships of mathematical ideas and the role math plays in other disciplines and life.

Standards for Mathematical Practice

- 1. **Make sense of problems and persevere in solving them:** Students will plan strategies to use and persevere in solving math problems.
- 2. **Reason abstractly and quantitatively:** Students will think about numbers in many ways and make sense of numerical relationships as they solve problems.
- 3. **Construct viable arguments and critique the reasoning of others:** Students will explain their thinking and make sense of the thinking of others.
- 4. **Model with mathematics:** Students will use representations to show their thinking in a variety of ways.
- 5. **Use appropriate tools strategically:** Students will use math tools such as tables, diagrams, and technology to explore and deepen their understanding of concepts.
- 6. **Attend to precision:** Students will use precise mathematical language and check their work for accuracy.
- 7. **Look for and make use of structure:** Students will use their current mathematical understandings to identify patterns and structure to make sense of new learning.
- 8. **Look for and express regularity in repeated reasoning:** Students will look for patterns and rules to help create general methods and shortcuts that can be applied to similar mathematical problems.

Quantitative Reasoning

Quantitative reasoning is the application of basic mathematics skills to analyze and process real-world information. In the K-5 grades, students use numbers, including written numerals, to represent quantities and to solve quantitative problems. Students will work on counting and cardinality, number and operations in Base Ten and fractions. Students will develop strategies to extend their understanding of the base ten system and apply those strategies to solve real-world problems using all four operations. Students progress from working with whole numbers to fractions and decimals.

In grades 6-8 students use reasoning about multiplication and division to solve ratio and rate problems about quantities. They develop an understanding of proportionality to solve problems and graph relationships. Overall, students extend and develop their understanding of rational numbers and can compute in all operations. Students use these operations to solve real-world problems. Students use this understanding of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

In the high school grades, the foundational concepts of operations with rational numbers and numerical properties built in the K-5 and 6-8 grade spans are applied to irrational numbers. Using a wider variety of units in modeling, (e.g. acceleration, currency conversions, and derived quantities such as person-hours and heating degree days), as well as the properties of rational and irrational numbers students are guided to the solution(s) to multi-step problems. Extending the properties of integer exponents to rational exponents deepens student understanding of how various but equivalent notations can facilitate their algebraic reasoning and problem-solving processes. Students are encouraged to expand these operations and properties into complex numbers, vectors, and matrices to further deepen their understanding of quantitative reasoning.

Strand	Quantitative Reasoning- Ratio and Proportional Relationships
Standard	QR.EA.1 Understand ratio and rate concepts and use ratio and rate reasoning to solve problems.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p>6.RP.A.1: Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. <i>For example, "The ratio of wings to beaks of the chickadees in the pine tree was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."</i></p> <p>6.RP.A.2: Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$ and use rate language in the context of a ratio relationship. <i>For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar." "We paid \$75 for 5 lobsters, which is a rate of \$15 per lobster." Expectations for unit rates in this grade are limited to non-complex fractions.</i></p> <p>6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</p> <p>6.RP.A.3a: Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</p> <p>6.RP.A.3b: Solve unit rate problems including those involving unit pricing and constant speed. <i>For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?</i></p> <p>6.RP.A.3c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $30/100$ times the quantity); solve problems involving finding the whole, given a part and the percent.</p> <p>6.RP.A.3d: Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</p>
Strand	Quantitative Reasoning- Ratio and Proportional Relationships
Standard	QR.EA.2 Analyze proportional relationships and use them to solve real-world and mathematical problems.
	Early Adolescence
	Grades 6-8

Performance Expectations	<p>7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. <i>For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour.</i></p> <p>7.RP.A.2: Recognize and represent proportional relationships between quantities.</p> <p>7.RP.A.2a: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.</p> <p>7.RP.A.2b: Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.</p> <p>7.RP.A.2c: Represent proportional relationships by equations. <i>For example, if the total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.</i></p> <p>7.RP.A.2d: Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.</p> <p>7.RP.A.3: Use proportional relationships to solve multistep ratio, rate, and percent problems. <i>Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</i></p>
Strand	Quantitative Reasoning- The Number System
Standard	QR.EA.3 Apply and extend previous understandings of operations with whole numbers to rational numbers.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p>6.NS.A.1: Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and/or equations to represent the problem. <i>For example, create a story context for $(\frac{2}{3}) \div (\frac{3}{4})$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(\frac{2}{3}) \div (\frac{3}{4}) = \frac{8}{9}$ because $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{2}{3}$. (In general, $(\frac{a}{b}) \div (\frac{c}{d}) = \frac{ad}{bc}$.)</i></p>

6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

7.NS.A.1a: Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has a zero charge because its two constituents are oppositely charged.*

7.NS.A.1b: Understand $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

7.NS.A.1c: Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts.

7.NS.A.1d: Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.A.2: Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

7.NS.A.2a: Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

7.NS.A.2b: Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

7.NS.A.2c: Apply properties of operations as strategies to multiply and divide rational numbers.

7.NS.A.2d: Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

7.NS.A.3: Solve real-world and mathematical problems involving the four operations with rational numbers. *Computations with rational numbers extend the rules for manipulating fractions to complex fractions.*

Strand	Quantitative Reasoning- The Number System
Standard	QR.EA.4 Compute fluently with multi-digit whole numbers and find common factors and multiples.

	Early Adolescence
	Grades 6-8
Performance Expectations	<p>6.NS.B.2: Fluently divide multi-digit numbers using the standard algorithm.</p> <p>6.NS.B.4: Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. (<i>For example: Use prime factorization to find the greatest common factor</i>); Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two relatively prime numbers. <i>For example, express $36 + 8$ as $4(9 + 2)$.</i></p>
Strand	Quantitative Reasoning - The Number System
Standard	QR.EA.5 Apply and extend previous understandings of numbers to the system of rational numbers.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p>6.NS.C.5: Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative rational numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.</p> <p>6.NS.C.6: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.</p> <p>6.NS.C.6a: Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.</p> <p>6.NS.C.6b: Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.</p> <p>6.NS.C.6c: Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.</p> <p>6.NS.C.7: Understand ordering and absolute value of rational numbers.</p>

	<p>6.NS.C.7a: Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. <i>For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.</i></p> <p>6.NS.C.7b: Write, interpret, and explain statements of order for rational numbers in real-world contexts. <i>For example, write $-3^{\circ} C > -7^{\circ} C$ to express the fact that $-3^{\circ} C$ is warmer than $-7^{\circ} C$.</i></p> <p>6.NS.C.7c: Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. <i>For example, for an account balance of -30 dollars, write $-30 = 30$ to describe the size of the debt in dollars.</i></p> <p>6.NS.C.7d: Distinguish comparisons of absolute value from statements about order. <i>For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.</i></p> <p>6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</p>
Strand	Quantitative Reasoning- The Number System
Standard	QR.EA.6 Know that there are numbers that are not rational, and approximate them by rational numbers.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p>8.NS.A.1: Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansions terminate in 0s or eventually repeats and convert a decimal expansion into a rational number.</p> <p>8.NS.A.2: Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). <i>For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.</i></p>

Algebraic Reasoning

Algebraic reasoning is about generalizing arithmetic operations and determining unknown quantities by recognizing and analyzing patterns along with developing generalizations about these patterns. In this K-5 strand, students explore, analyze, represent, and generalize mathematical ideas and relationships. Students will develop an understanding of the fundamental properties of number and operations, understand the use of the equal sign to represent equivalence, and use quantitative reasoning to understand mathematical relationships.

Students in grades 6-8 progress in their understanding of variables in mathematical expressions and equations. They understand that expressions in different forms can be equivalent, use the properties of operations to rewrite expressions in equivalent forms, and describe relationships between quantities. Students begin to analyze and solve real-world and mathematical problems using equations and inequalities. They construct and interpret tables and graphs. Understanding builds from writing and solving simple equations to solving proportional situations. These skills lead to exploring slope and y-intercept and relationships between variables, and eventually include multiple equations to solve systems of linear equations. Students grow to understand that the concept of a function is a rule that assigns one output to each input, and they learn to translate among different representations of functions.

In grades 9-12, students will continue to develop their understanding of expressions, equations, functions and function notation. They will interpret the structure of algebraic expressions and be able to write expressions in equivalent forms to reveal information and to solve problems. Students will perform arithmetic operations on polynomials and rewrite rational functions. An understanding of the relationship between zeros and factors of polynomials will transition into using polynomial identities to solve problems. Students will create equations that describe relationships and solve equations as a process of reasoning (with appropriate justification). They will represent and solve equations, inequalities, and systems of equations using a variety of mathematically sound techniques.

Students will interpret functions that arise in applications in terms of context and analyze functions using different representations. They will build functions that model relationships between two quantities, and build new functions from existing functions through transformations, combinations, compositions, and examining the inverse. Students will construct and compare linear, quadratic, and exponential models and use those models to solve problems. They will interpret expressions for functions in terms of the situation they model. Students will be encouraged to extend their understanding of algebra and functions and apply similar processes of reasoning to polynomial, logarithmic and trigonometric functions and their graphs.

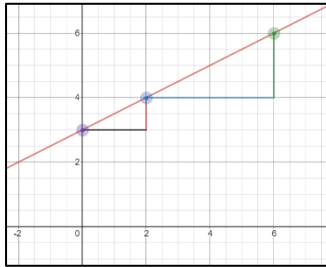
Strand	Algebraic Reasoning - Expressions and Equations
Standard	AR.EA.1 Apply and extend previous understandings of arithmetic to algebraic expressions.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p>6.EE.A.1: Write and evaluate numerical expressions involving whole-number exponents.</p> <p>6.EE.A.2: Write, read, and evaluate expressions in which letters represent numbers.</p> <p>6.EE.A.2a: Write expressions that record operations with numbers and with letters representing numbers. <i>For example, express the calculation "Subtract y from 5" as $5 - y$.</i></p> <p>6.EE.A.2b: Identify parts of an expression using mathematical terms (including but not limited to: sum, term, product, factor, quotient, coefficient, variable, constant); view one or more parts of an expression as a single entity. <i>For example, describe the expression $2(x + 7)$ as a product of two factors; view $(x + 7)$ as both a single entity and a sum of two terms.</i></p> <p>6.EE.A.2c: Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, using the order of operations. <i>For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.</i></p> <p>6.EE.A.3: Apply the properties of operations to generate equivalent expressions. <i>For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to factor the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.</i></p> <p>6.EE.A.4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). <i>For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.</i></p>
Strand	Algebraic Reasoning - Expressions and Equations
Standard	AR.EA.2 Reason about and solve one-variable equations and inequalities.
	Early Adolescence
	Grades 6-8

Performance Expectations	<p>6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.</p> <p>6.EE.B.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</p> <p>6.EE.B.7: Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. <i>For example, Sal is paid \$0.50 per pound of blueberries that she rakes. If she rakes x pounds, and earns \$17.25, write and solve an equation that determines how many pounds she raked.</i></p> <p>6.EE.B.8: Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.</p>
Strand	Algebraic Reasoning - Expressions and Equations
Standard	AR.EA.3 Represent and analyze quantitative relationships between dependent and independent variables.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p>6.EE.C.9: Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables and relate these to the equation. <i>For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.</i></p>
Strand	Algebraic Reasoning - Expressions and Equations
Standard	AR.EA.4 Use properties of operations to generate equivalent expressions.
	Early Adolescence
	Grades 6-8

Performance Expectations	<p>7.EE.A.1: Apply properties of operations to add, subtract, factor, and expand linear expressions with rational coefficients. <i>For example, $4x + 2 = 2(2x+1)$ and $-3(x-5/3) = -3x + 5$</i></p> <p>7.EE.A.2: Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. <i>For example, A shirt is on sale for 20% off the regular price, p. The discount can be expressed as $0.2p$. The new price for the shirt can be expressed as $p - 0.2p$ or $0.8p$.</i></p>
Strand	Algebraic Reasoning - Expressions and Equations
Standard	AR.EA.5 Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p>7.EE.B.3: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. <i>For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.</i></p> <p>7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities.</p> <p>7.EE.B.4a: Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. <i>For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?</i></p> <p>7.EE.B.4b: Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. <i>For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make and describe the solutions.</i></p>

Strand	Algebraic Reasoning - Expressions and Equations
Standard	AR.EA.6 Work with radicals and integer exponents.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p>8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. <i>For example, $3^2 \times 3^{-5} = 3^{-3} = (1/3)^3 = 1/27$.</i></p> <p>8.EE.A.2: Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.</p> <p>8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. <i>For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger.</i></p> <p>8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.</p>
Strand	Algebraic Reasoning - Expressions and Equations
Standard	AR.EA.7 Understand the connections between proportional relationships, lines, and linear equations.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p>8.EE.B.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. <i>For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</i></p> <p>8.EE.B.6: Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the</p>

equation $y = mx + b$ for a line intercepting the vertical axis at b . For example, given the line $y = 0.5x + 3$ explain why the similar triangles have the same slope.



Strand	Algebraic Reasoning - Expressions and Equations
Standard	AR.EA.8 Analyze and solve linear equations and pairs of simultaneous linear equations.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p>8.EE.C.7: Solve linear equations in one variable.</p> <p>8.EE.C.7a: Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).</p> <p>8.EE.C.7b: Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.</p> <p>8.EE.C.8: Analyze and solve pairs of simultaneous linear equations.</p> <p>8.EE.C.8a: Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</p> <p>8.EE.C.8b: Solve systems of two linear equations in two variables algebraically (i.e. by substitution or elimination) and estimate solutions by graphing the equations. Solve simple cases by inspection. <i>For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.</i></p>

	8.EE.C.8c: Solve real-world and mathematical problems leading to two linear equations in two variables. <i>For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.</i>
Strand	Algebraic Reasoning - Functions
Standard	AR.EA.9 Define, evaluate, and compare functions in order to model relationships between quantities.
	Early Adolescence
	Grades 6-8 Function notation is not required for Grade 8.
Performance Expectations	<p>8.F.A.1: Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.</p> <p>8.F.A.2: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.</i></p> <p>8.F.A.3: Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. <i>For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.</i></p> <p>8.F.B.4: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p> <p>8.F.B.5: Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</p>

Geometric Reasoning

Geometric reasoning is the use of critical thinking, logical argument and spatial reasoning to solve problems and find new relationships. Students must first have a critical understanding of any underlying assumptions and relationships. This allows them to develop coherent knowledge and apply their reasoning skills. In this K-5 strand, students will develop an understanding of the attributes of two- and three-dimensional shapes and apply this knowledge to real-world problems. Students will also be introduced to the coordinate system.

Students in grades 6-8 work with two- and three-dimensional objects to reason about relationships among shapes. They learn to calculate area, surface area, volume, and circumference using multiple methods including decomposing shapes so that they can develop, justify, and use formulas including the Pythagorean Theorem and its converse. They use scale drawings and informal constructions to gain familiarity with the relationships between angles formed by intersecting lines and transformations.

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more complex definitions and reasoning of proofs. Students make geometric constructions using a variety of technological tools and connect these explorations to reasoning and proofs. Attributes of parallel lines intersected by a transversal are further developed and extended into properties of triangles, quadrilaterals, and regular polygons as well as circles using informal and formal reasoning. Fundamental to the concepts of congruence, similarity, and symmetry are transformations which can preserve distance and angles.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity. The Pythagorean Theorem along with these ratios are fundamental in many real-world and theoretical situations. Correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. Concepts of two- and three-dimensional shapes are explored using algebraic formulas and modeling. Students are encouraged to extend their geometric reasoning through the exploration of trigonometric identities and properties of conic sections.

Strand	Geometric Reasoning - Geometry
Standard	GR.EA.1 Solve real-world and mathematical problems involving angle measure, area, surface area, and volume.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p>6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.</p> <p>6.G.A.2: Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l w h$ and $V = B h$ (where B stands for the area of the base) to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.</p> <p>6.G.A.3: Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.</p> <p>6.G.A.4: Represent three-dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.</p> <p>7.G.B.4: Know that a circle is a two-dimensional shape created by connecting all the points equidistant from a fixed point called the center of the circle. Understand and describe the relationships among the radius, diameter, circumference and area of a circle. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.</p> <p>7.G.B.5: Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.</p> <p>7.G.B.6: Solve real-world and mathematical problems involving area, volume and surface area of two- and/or three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</p>

	8.G.C.9: Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
Strand	Geometric Reasoning - Geometry
Standard	GR.EA.2 Draw, construct, and describe geometrical figures and describe the relationships between them.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p>7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.</p> <p>7.G.A.2: Draw (freehand, with ruler and protractor, and with technology) two-dimensional geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.</p> <p>7.G.A.3: Describe the shape of the cross-section two-dimensional face of the figures that results from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.</p>
Strand	Geometric Reasoning - Geometry
Standard	GR.EA.3 Understand congruence and similarity using physical models, transparencies, or geometry software,
	Early Adolescence
	Grades 6-8
Performance Expectations	<p>8.G.A.1: Verify experimentally the properties of rotations, reflections, and translations: 8.G.A.1a: Lines are taken to lines, and line segments to line segments of the same length. 8.G.A.1b: Angles are taken to angles of the same measure. 8.G.A.1c: Parallel lines are taken to parallel lines.</p> <p>8.G.A.2: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.</p> <p>8.G.A.3: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.</p>

	<p>8.G.A.4: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.</p> <p>8.G.A.5: Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. <i>For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.</i></p>
Strand	Geometric Reasoning - Geometry
Standard	GR.EA.4 Understand and apply the Pythagorean Theorem.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p>8.G.B.6: Explain a proof of the Pythagorean Theorem and its converse using pictures, diagrams, narratives or models.</p> <p>8.G.B.7: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</p> <p>8.G.B.8: Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</p>

Statistical Reasoning

Statistical reasoning is the way people analyze data and make sense of information. It involves generalizations that connect one concept to another. In this K-5 strand, students will develop strategies to represent and interpret data, describe and compare measurable attributes, and understand concepts of measurement including perimeter, area, volume, time, and money.

Students in grades 6-8 continue to develop their ability to think statistically. Measures of central tendency (mean, median, and mode) as well as measures of variability (range, interquartile range, mean absolute deviation) are used to describe data. Previous work with single data distributions is expanded to compare two data distributions and address questions about differences between populations. Informal work with random sampling and learning about the importance of representative samples for drawing inferences is introduced. Students then expand their statistical understanding to include connections involving modeling with linear equations, as well as non-linear expressions. Looking for patterns in a bivariate data system is emphasized.

In grades 9-12 students extend their statistical understanding of univariate and bi-variate data in a real-world context. This understanding is used to make decisions or predictions based on the data. Since data can be variable, statistics provide the tools for taking this variability into account. Data can be categorical or quantitative in nature. Appropriate methods for collecting, displaying, summarizing, and analyzing data are learned and employed. Algebraic and geometric reasoning are utilized to create linear regression models in order to interpret the relationship between two quantitative variables when appropriate.

The conditions under which data are collected and the use of randomization in the design of a study are necessary for drawing valid conclusions about the population under study. Since random processes can be described mathematically by using a probability model, the role of probability in making predictions or in making decisions becomes evident. Technology makes it possible to generate plots, find regression functions, compute correlation coefficients, and run simulations to better understand data. Statistical reasoning is a deeply rich and complex process which is essential to comprehend in order to stay informed in civic matters and personal decision-making.

Strand	Statistical Reasoning - Statistics & Probability
Standard	SR.EA.1 Summarize distribution using measures of center, variability, and graphical displays.
	Early Adolescence
	Grades 6-8
Performance Expectations	<p>6.SP.A.1: Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. <i>For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.</i></p> <p>6.SP.A.2: Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center (mean, median and/or mode), spread (range and/or interquartile range), and overall shape.</p> <p>6.SP.A.3: Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.</p> <p>6.SP.B.4: Display numerical data in plots on a number line, including dot plots, histograms, and box plots.</p> <p>6.SP.B.5: Summarize numerical data sets in relation to their context, such as by:</p> <p>6.SP.B.5a: Reporting the number of observations.</p> <p>6.SP.B.5b: Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.</p> <p>6.SP.B.5c: Calculating quantitative measures of center (median and/or mean) and variability (range and/or interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.</p> <p>6.SP.B.5d: Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.</p>
Strand	Statistical Reasoning - Statistics & Probability
Standard	SR.EA.2 Use random sampling, visual representations, and measures of center and variability to draw inferences about one or more populations.
	Early Adolescence

	Grades 6-8
Performance Expectations	<p>7.SP.A.1: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.</p> <p>7.SP.A.2: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. <i>For example, estimate the mean length of a largemouth bass in a lake by randomly sampling largemouth bass from the lake; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.</i></p> <p>7.SP.B.3: Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. <i>For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team and both distributions have similar variability (mean absolute deviation) of about 5 cm. The difference between the mean heights of the two teams (10 cm) is about twice the variability (5 cm mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.</i></p> <p>7.SP.B.4: Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. <i>For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.</i></p>
Strand	Statistical Reasoning - Statistics & Probability
Standard	SR.EA.3 Investigate chance processes and develop, use, and evaluate probability models.
	Early Adolescence
	Grades 6-8
Performance Expectations	7.SP.C.5: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

	<p>7.SP.C.6: Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. <i>For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.</i></p> <p>7.SP.C.7: Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.</p> <p>7.SP.C.7a: Develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events. <i>For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.</i></p> <p>7.SP.C.7b: Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. <i>For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?</i></p> <p>7.SP.C.8: Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.</p> <p>7.SP.C.8a: Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.</p> <p>7.SP.C.8b: Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.</p> <p>7.SP.C.8c: Design and use a simulation to generate frequencies for compound events. <i>For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?</i></p>
Strand	Statistical Reasoning - Statistics & Probability
Standard	SR.EA.4 Investigate patterns of association in bivariate data.
	Early Adolescence
	Grades 6-8
Performance Expectations	8.SP.A.1: Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.A.2: Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

8.SP.A.3: Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

8.SP.A.4: Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Definitions:

Strand: A body of knowledge in a content area identified by a simple title.

Standard: Enduring understandings and skills that students can apply and transfer to contexts that are new to the student.

Performance Expectation: Building blocks to the standard and measurable articulations of what the student understands and can do.