Mathematics Standards Introduction

A strong mathematics education depends upon a clear understanding of its interrelated concepts, skills and practices to ensure students are on the pathway to success in their academic careers. The knowledge and skills students need to be prepared for mathematics in college, career, and life are woven throughout the K-12 mathematics performance expectations.

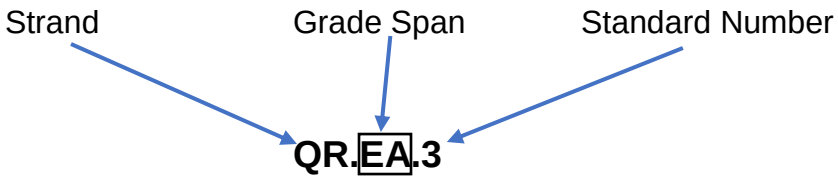
Outline of Mathematics Strands and Standards

These mathematical performance expectations are building blocks to standards. The standards are grouped into four strands:

- **Quantitative Reasoning** (Blue)**:** Counting and Cardinality, Number and Operations in Base Ten, Number and Operations Fractions, Ratio and Proportional Relations, The Number System, and Number and Quantity.
- **Algebraic Reasoning** (Green)**:** Operations and Algebraic Thinking, Expressions and Equations, Functions, and Algebra
- **Geometric Reasoning** (Red)**:** Geometry
- **Statistical Reasoning** (Purple)**:** Measurement and Data, Statistics and Probability

These mathematical performance expectations are broken into three grade spans: **C**hildhood (K-5), **E**arly **A**dolescence (6-8), and **A**dolescence (9-Diploma). The strands are color-coded, as indicated above, for continuity throughout the grade spans. Standards do not work in isolation, they are connected through and across strands.

How to Read the Standards



Within the high school performance expectations, modeling is woven throughout the four strands and is denoted with a star (★). The high school standards also contain some performance expectations which are denoted by a plus (**+**). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.

The Guiding Principles & Standards for Mathematical Practice

The Guiding Principles influence education in Maine and should be reflected throughout Mathematics curriculum. The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Full descriptions of the Guiding Principles and Standards for Mathematical Practice can be found in the Supplemental Material. Examples of how students can show evidence of those **Guiding Principles** and **Standards for Mathematical Practice** **may** include:

Guiding Principles

A. **A clear and effective communicator:** Students will use written, oral, symbolic, and visual forms of expression to communicate mathematically.

B. **A self-directed and lifelong learner:** Students generate and persevere in solving questions while demonstrating a growth mindset.

C. **A creative and practical problem solver:** Students will pose and solve mathematical problems by using a variety of strategies that connect to real-world examples.

D. **A responsible and involved citizen:** Students make sense of the world around them through mathematics including economic literacy.

E. **An integrative and informed thinker:** Students connect mathematics to other learning by understanding the interrelationships of mathematical ideas and the role math plays in other disciplines and life.

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them:** Students will plan strategies to use and persevere in solving math problems.

2. **Reason abstractly and quantitatively:** Students will think about numbers in many ways and make sense of numerical relationships as they solve problems.

3. **Construct viable arguments and critique the reasoning of others:** Students will explain their thinking and make sense of the thinking of others.

4. **Model with mathematics:** Students will use representations to show their thinking in a variety of ways.

5. **Use appropriate tools strategically:** Students will use math tools such as tables, diagrams, and technology to explore and deepen their understanding of concepts.

6. **Attend to precision:** Students will use precise mathematical language and check their work for accuracy.

7. **Look for and make use of structure:** Students will use their current mathematical understandings to identify patterns and structure to make sense of new learning.

8. **Look for and express regularity in repeated reasoning:** Students will look for patterns and rules to help create general methods and shortcuts that can be applied to similar mathematical problems.

Quantitative Reasoning

Quantitative reasoning is the application of basic mathematics skills to analyze and process real-world information. In the K-5 grades, students use numbers, including written numerals, to represent quantities and to solve quantitative problems. Students will work on counting and cardinality, number and operations in Base Ten and fractions. Students will develop strategies to extend their understanding of the base ten system and apply those strategies to solve real-world problems using all four operations. Students progress from working with whole numbers to fractions and decimals.

In grades 6-8 students use reasoning about multiplication and division to solve ratio and rate problems about quantities. They develop an understanding of proportionality to solve problems and graph relationships. Overall, students extend and develop their understanding of rational numbers and can compute in all operations. Students use these operations to solve real-world problems. Students use this understanding of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

In the high school grades, the foundational concepts of operations with rational numbers and numerical properties built in the K-5 and 6-8 grade spans are applied to irrational numbers. Using a wider variety of units in modeling, (e.g. acceleration, currency conversions, and derived quantities such as person-hours and heating degree days), as well as the properties of rational and irrational numbers students are guided to the solution(s) to multi-step problems. Extending the properties of integer exponents to rational exponents deepens student understanding of how various but equivalent notations can facilitate their algebraic reasoning and problem-solving processes. Students are encouraged to expand these operations and properties into complex numbers, vectors, and matrices to further deepen their understanding of quantitative reasoning.

Strand	**Quantitative Reasoning- Counting and Cardinality**		
Standard	**QR.C.1** Know the number names and the count sequence.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations	**K.CC.A.1:** Count to 100 by ones and by tens **K.CC.A.2:** Count forward beginning from a given number within the known sequence (instead of having to begin at 1) **K.CC.A.3:** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects)		
Strand	**Quantitative Reasoning- Counting and Cardinality**		
Standard	**QR.C.2** Count to tell the number of objects.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations	**K.CC.B.4:** Understand the relationship between numbers and quantities; connect counting to cardinality. **K.CC.B.4a:** When counting objects, say the number names in the standard order, pairing each object with one and only one number name		

	and each number name with one and only one object. **K.CC.B.4b:** Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. **K.CC.B.4c:** Understand that each successive number name refers to a quantity that is one larger. Recognize the one more pattern of counting using objects. **K.CC.B.5:** Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.		
Strand	**Quantitative Reasoning- Counting and Cardinality**		
Standard	**QR.C.3** Compare numbers.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations	**K.CC.C.6:** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another		

	group, e.g., by using matching and counting strategies. Include groups with up to ten objects. **K.CC.C.7:** Compare two numbers between 1 and 10 presented as written numerals.		
Strand	**Quantitative Reasoning - Numbers and Operations in Base Ten**		
Standard	**QR.C.4** Extend the counting sequence.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations		**1.NBT.A.1:** Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	
Strand	**Quantitative Reasoning - Numbers and Operations in Base Ten**		
Standard	**QR.C.5** Understand place value.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations	**K.NBT.A.1:** Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8 and 10+8=18); understand that these numbers are composed of	**1.NBT.B.2:** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: **1.NBT.B.2a:** 10 can be thought of as a bundle of ten ones - called a “ten.”	**2.NBT.A.1:** Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: **2.NBT.A.1a:** 100 can be thought of as a bundle of ten tens - called a “hundred.”

	ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	**1.NBT.B.2b:** The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. **1.NBT.B.2c:** The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). **1.NBT.B.3:** Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	**2.NBT.A.1b:** The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). **2.NBT.A.2:** Count within 1000; skip-count by 5s, 10s, and 100s. Identify patterns in skip counting at any number. (For example, 37, 47, 57 or 328, 428, 528, etc.) **2.NBT.A.3:** Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. **2.NBT.A.4:** Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
Strand	**Quantitative Reasoning - Numbers and Operations in Base Ten**		
Standard	**QR.C.6** Use place value understanding and properties of operations to add and subtract.		
	Childhood		
	Kindergarten	Grade 1	Grade 2

Performance Expectations			**2.NBT.B.5:** Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
		1.NBT.C.4: Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	**2.NBT.B.7:** Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
		1.NBT.C.5: Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	**2.NBT.B.8:** Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
			2.NBT.B.6: Add up to four two-digit numbers using strategies based on place value and properties of operations.

		1.NBT.C.6: Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	**2.NBT.B.9:** Explain why addition and subtraction strategies work, using place value and the properties of operations. Explanations may be supported by drawings or objects.
Strand	**Quantitative Reasoning - Numbers and Operations in Base Ten**		
Standard	**QR.C.7** Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers and decimals to hundredths.		
	Childhood		
	Grade 3 A range of algorithms may be used.	Grade 4 Grade 4 expectations in this *strand* are limited to whole numbers less than or equal to 1,000,000	Grade 5
Performance Expectations	**3.NBT.A.2:** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	**4.NBT.B.4:** Fluently add and subtract multi-digit whole numbers using the standard algorithm.	**5.NBT.B.5:** Fluently multiply multi-digit whole numbers using the standard algorithm.

	3.NBT.A.3: Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.	**4.NBT.B.5:** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **4.NBT.B.6:** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	**5.NBT.B.6:** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **5.NBT.B.7:** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, money and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
Strand	**Quantitative Reasoning - Numbers and Operations in Base Ten**		
Standard	**QR.C.8** Understand the place value system.		
	Childhood		
	Grade 3	Grade 4	Grade 5

	A range of algorithms may be used.	Grade 4 expectations in this *strand* are limited to whole numbers less than or equal to 1,000,000	
Performance Expectations	**3.NBT.A.1:** Use place value understanding to round whole numbers to the nearest 10 or 100.	**4.NBT.A.3:** Use place value understanding to round multi-digit whole numbers to any place. **4.NBT.A.2:** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **4.NBT.A.1:** Recognize that in a multi-digit whole number, a digit in any place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division*.	**5.NBT.A.4:** Use place value understanding to round decimals to any place. **5.NBT.A.3:** Read, write, and compare decimals to thousandths. **5.NBT.A.3a:** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). **5.NBT.A.3b:** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **5.NBT.A.1:** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. **5.NBT.A.2:** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.

			Use whole-number exponents to denote powers of 10.
Strand	**Quantitative Reasoning - Numbers and Operations: Fractions**		
Standard	**QR.C.9** Develop and extend the understanding of fractions as numbers, including equivalence and ordering.		
	Childhood		
	Grade 3 Grade 3 expectations in this *strand* are limited to fractions with denominators 2, 3, 4, 6, and 8.	Grade 4 Grade 4 expectations in this *strand* are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	Grade 5
Performance Expectations	**3.NF.A.1:** Understand a unit fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*. **3.NF.A.2:** Understand a fraction as a number on the number line; represent fractions on a number line diagram. **3.NF.A.2a:** Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.		

	3.NF.A.2b: Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line. **3.NF.A.3:** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **3.NF.A.3a:** Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. **3.NF.A.3b:** Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model. **3.NF.A.3c:** Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram*.	**4.NF.A.1:** Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions, including fractions greater than 1.	

	3.NF.A.3d: Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	**4.NF.A.2:** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	
Strand	**Quantitative Reasoning - Numbers and Operations: Fractions**		
Standard	**QR.C.10** Understand decimal notation for fractions, and compare decimal fractions.		
	Childhood		
	Grade 3	Grade 4 Grade 4 expectations in this *strand* are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with	Grade 5

		unlike denominators in general is not a requirement at this grade.	
Performance Expectations		**4.NF.C.5:** Express a fraction with denominator 10 as an equivalent fraction with denominator 100 and use this technique to add two fractions with respective denominators 10 and 100.[2] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100*. **4.NF.C.6:** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram*. **4.NF.C.7:** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	
Strand	**Quantitative Reasoning - Numbers and Operations: Fractions**		
Standard	**QR.C.11** Use equivalent fractions as a strategy to add and subtract fractions.		
	Childhood		
	Grade 3	Grade 4	Grade 5

		Grade 4 expectations in this *strand* are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.	
Performance Expectations		**4.NF.B.3:** Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. **4.NF.B.3a:** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. **4.NF.B.3b:** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model to build fractions from unit fractions. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8*. **4.NF.B.3c:** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction,	**5.NF.A.1:** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as

		and/or by using properties of operations and the relationship between addition and subtraction. **4.NF.B.3d:** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* **5.NF.A.2:** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2*.
Strand	**Quantitative Reasoning - Numbers and Operations: Fractions**		
Standard	**QR.C.12** Apply and extend previous understandings of multiplication and division to multiply and divide fractions.		
	Childhood		
	Grade 3	Grade 4	Grade 5 Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.
Performance Expectations		**4.NF.B.4:** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.	**5.NF.B.4:** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

		4.NF.B.4a: Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4)*.	**5.NF.B.4a:** Interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = (ac)/(bd).*
		4.NF.B.4b: Understand a multiple of a/b as a multiple of 1/b and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*	**5.NF.B.4b:** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas.
		4.NF.B.4c: Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed?*	**5.NF.B.3:** Interpret a fraction as division of the numerator by the denominator (*a*/*b* = *a* ÷ *b*). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size*

		Between what two whole numbers does your answer lie?	*3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* **5.NF.B.5:** Interpret multiplication scaling (resizing), by: **5.NF.B.5a:** Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. **5.NF.B.5b:** Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. **5.NF.B.6:** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

			5.NF.B.7: Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1] **5.NF.B.7a:** Interpret division of a unit fraction by a non-zero whole number and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3*. **5.NF.B.7b:** Interpret division of a whole number by a unit fraction and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4*. **5.NF.B.7c:** Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Algebraic Reasoning

Algebraic reasoning is about generalizing arithmetic operations and determining unknown quantities by recognizing and analyzing patterns along with developing generalizations about these patterns. In this K-5 strand, students explore, analyze, represent, and generalize mathematical ideas and relationships. Students will develop an understanding of the fundamental properties of number and operations, understand the use of the equal sign to represent equivalence, and use quantitative reasoning to understand mathematical relationships.

Students in grades 6-8 progress in their understanding of variables in mathematical expressions and equations. They understand that expressions in different forms can be equivalent, use the properties of operations to rewrite expressions in equivalent forms, and describe relationships between quantities. Students begin to analyze and solve real-world and mathematical problems using equations and inequalities. They construct and interpret tables and graphs. Understanding builds from writing and solving simple equations to solving proportional situations. These skills lead to exploring slope and y-intercept and relationships between variables, and eventually include multiple equations to solve systems of linear equations. Students grow to understand that the concept of a function is a rule that assigns one output to each input, and they learn to translate among different representations of functions.

In grades 9-12, students will continue to develop their understanding of expressions, equations, functions and function notation. They will interpret the structure of algebraic expressions and be able to write expressions in equivalent forms to reveal information and to solve problems. Students will perform arithmetic operations on polynomials and rewrite rational functions. An understanding of the relationship between zeros and factors of polynomials will transition into using polynomial identities to solve problems. Students will create equations that describe relationships and solve equations as a process of reasoning (with appropriate justification). They will represent and solve equations, inequalities, and systems of equations using a variety of mathematically sound techniques.

Students will interpret functions that arise in applications in terms of context and analyze functions using different representations. They will build functions that model relationships between two quantities, and build new functions from existing functions through transformations, combinations, compositions, and examining the inverse. Students will construct and compare linear, quadratic, and exponential models and use those models to solve problems. They will interpret expressions for functions in terms of the situation they model. Students will be encouraged to extend their understanding of algebra and functions and apply similar processes of reasoning to polynomial, logarithmic and trigonometric functions and their graphs.

Strand	**Algebraic Reasoning – Operations and Algebraic Thinking**		
Standard	**AR.C.1** Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.		
	Childhood		
	Kindergarten (Drawings need not show detail but should show the mathematics in the problem. This applies wherever drawings are mentioned in the Standards.)	Grade 1	Grade 2
Performance Expectations	**K.OA.A.1:** Represent addition and subtraction with objects, fingers, mental images, drawings sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. **K.OA.A.2:** Solve addition and subtraction word problems, and add and subtract within 10, (e.g., by using objects or drawings to represent the problem).	**1.OA.A.1:** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	**2.OA.A.1:** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

	K.OA.A.3: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). **K.OA.A.4:** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. **K.OA.A.5:** Fluently add and subtract within 5 including zero.	**1.OA.A.2:** Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.)	
Strand	**Algebraic Reasoning – Operations and Algebraic Thinking**		

Standard	**AR.C.2** Understand and apply properties of operation and the relationship between addition and subtraction within 20.		
	Childhood		
	Kindergarten	Grade 1 Students need not use formal terms for these properties.	Grade 2
Performance Expectations		**1.OA.B.3:** Apply properties of operations as strategies to add. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.) a + 0 = a (Additive identity property of 0)* **1.OA.B.4:** Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.* **1.OA.C.5:** Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). **1.OA.C.6:** Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.	**2.OA.B.2:** Fluently add and subtract within 20 using mental strategies. By end of Grade 2,

		Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). **1.OA.D.7:** Understand the meaning of the equal sign and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2. **1.OA.D.8:** Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ - 3, 6 + 6 = _.*	know from memory all sums of two one-digit numbers.
Strand	**Algebraic Reasoning – Operations and Algebraic Thinking**		

Standard	**AR.C.3** Work with equal groups of objects to gain foundations for multiplication.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations			**2.OA.C.3:** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. **2.OA.C.4:** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
Strand	**Algebraic Reasoning – Operations and Algebraic Thinking**		
Standard	**AR.C.4** Understand properties of multiplication and the relationship between multiplication and division to represent and solve problems within 100.		
	Childhood		
	Grade 3 Students need not use formal terms for these properties.	Grade 4	Grade 5
Performance Expectations	**3.OA.A.1:** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total		

	number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7*. **3.OA.A.2:** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8*. **3.OA.A.3:** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.		

	3.OA.A.4: Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?* **3.OA.B.5:** Apply properties of operations as strategies to multiply. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* **3.OA.B.6:** Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the*		

	number that makes 32 when multiplied by 8. **3.OA.C.7:** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.		
Strand	**Algebraic Reasoning – Operations and Algebraic Thinking**		
Standard	**AR.C.5** Solve problems involving the four operations.		
	Childhood		
	Grade 3 This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in conventional order when there are no parentheses to specify a particular order (Order of Operations).	Grade 4	Grade 5
Performance Expectations		**4.OA.A.1:** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	

	3.OA.D.8: Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	**4.OA.A.2:** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. **4.OA.A.3:** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	
Strand	**Algebraic Reasoning – Operations and Algebraic Thinking**		
Standard	**AR.C.6** Gain familiarity with factors and multiples.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations		**4.OA.B.4:** Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is	

		a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	
Strand	**Algebraic Reasoning – Operations and Algebraic Thinking**		
Standard	**AR.C.7** Write and interpret numerical expressions.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations			**5.OA.A.1:** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. **5.OA.A.2:** Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*
Strand	**Algebraic Reasoning – Operations and Algebraic Thinking**		
Standard	**AR.C.8** Identify, explain, generate and analyze patterns.		

	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations	**3.OA.D.9:** Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends*.	**4.OA.C.5:** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way*.	**5.OA.B.3:** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so*.

Geometric Reasoning

Geometric reasoning is the use of critical thinking, logical argument and spatial reasoning to solve problems and find new relationships. Students must first have a critical understanding of any underlying assumptions and relationships. This allows them to develop coherent knowledge and apply their reasoning skills. In this K-5 strand, students will develop an understanding of the attributes of two- and three-dimensional shapes and apply this knowledge to real-world problems. Students will also be introduced to the coordinate system.

Students in grades 6-8 work with two- and three-dimensional objects to reason about relationships among shapes. They learn to calculate area, surface area, volume, and circumference using multiple methods including decomposing shapes so that they can develop, justify, and use formulas including the Pythagorean Theorem and its converse. They use scale drawings and informal constructions to gain familiarity with the relationships between angles formed by intersecting lines and transformations.

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more complex definitions and reasoning of proofs. Students make geometric constructions using a variety of technological tools and connect these explorations to reasoning and proofs. Attributes of parallel lines intersected by a transversal are further developed and extended into properties of triangles, quadrilaterals, and regular polygons as well as circles using informal and formal reasoning. Fundamental to the concepts of congruence, similarity, and symmetry are transformations which can preserve distance and angles.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity. The Pythagorean Theorem along with these ratios are fundamental in many real-world and theoretical situations. Correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. Concepts of two- and three-dimensional shapes are explored using algebraic formulas and modeling. Students are encouraged to extend their geometric reasoning through the exploration of trigonometric identities and properties of conic sections.

Strand	**Geometric Reasoning - Geometry**		
Standard	**GR.C.1** Identify, describe, analyze, compare, create, and compose shapes based on their attributes.		
	Childhood		
	Kindergarten	Grade 1 Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.	Grade 2 Sizes are compared directly or visually, not compared by measuring.
Performance Expectations	**K.G.A.1:** Describe objects in the environment using names of shapes and describe the relative positions of these objects using terms such as *above*, *below*, *beside, in front of*, *behind*, and *next to*. **K.G.A.2:** Correctly name shapes regardless of their orientations or overall size. **K.G.A.3:** Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid"). **K.G.B.4:** Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal	**1.G.A.1:** Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	**2.G.A.1:** Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals (including squares, rectangles, rhombuses, and trapezoids) pentagons, hexagons, and cubes. Sizes are compared directly or visually, not compared by measuring.

	language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). **K.G.B.5:** Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. **K.G.B.6:** Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle*?"	**1.G.A.2:** Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. **1.G.A.3:** Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*.	**2.G.A.2:** Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. **2.G.A.3:** Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two

		Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.
Strand	**Geometric Reasoning - Geometry**		
Standard	**GR.C.2** Analyze, compare, create, and compose shapes based on their attributes.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations	**3.G.A.1:** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. **3.G.A.2:** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each*		**5.G.B.3:** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

	part as 1/4 of the area of the shape.		**5.G.B.4:** Classify two-dimensional figures in a hierarchy based on properties. (e.g., all rectangles are parallelograms, because they are all quadrilaterals with two pairs of opposite sides parallel.)
Strand	**Geometric Reasoning - Geometry**		
Standard	**GR.C.3** Draw and identify lines and angles and classify shapes by properties of their lines and angles.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations		**4.G.A.1:** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. **4.G.A.2:** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category and identify right triangles. **4.G.A.3:** Recognize a line of symmetry for a two-dimensional	

		figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	
Strand	**Geometric Reasoning - Geometry**		
Standard	**GR.C.4** Graph points on the coordinate plane to solve real-world and mathematical problems.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations			**5.G.A.1:** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

			5.G.A.2: Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.

Statistical Reasoning

Statistical reasoning is the way people analyze data and make sense of information. It involves generalizations that connect one concept to another. In this K-5 strand, students will develop strategies to represent and interpret data, describe and compare measurable attributes, and understand concepts of measurement including perimeter, area, volume, time, and money.

Students in grades 6-8 continue to develop their ability to think statistically. Measures of central tendency (mean, median, and mode) as well as measures of variability (range, interquartile range, mean absolute deviation) are used to describe data. Previous work with single data distributions is expanded to compare two data distributions and address questions about differences between populations. Informal work with random sampling and learning about the importance of representative samples for drawing inferences is introduced. Students then expand their statistical understanding to include connections involving modeling with linear equations, as well as non-linear expressions. Looking for patterns in a bivariate data system is emphasized.

In grades 9-12 students extend their statistical understanding of univariate and bi-variate data in a real-world context. This understanding is used to make decisions or predictions based on the data. Since data can be variable, statistics provide the tools for taking this variability into account. Data can be categorical or quantitative in nature. Appropriate methods for collecting, displaying, summarizing, and analyzing data are learned and employed. Algebraic and geometric reasoning are utilized to create linear regression models in order to interpret the relationship between two quantitative variables when appropriate.

The conditions under which data are collected and the use of randomization in the design of a study are necessary for drawing valid conclusions about the population under study. Since random processes can be described mathematically by using a probability model, the role of probability in making predictions or in making decisions becomes evident. Technology makes it possible to generate plots, find regression functions, compute correlation coefficients, and run simulations to better understand data. Statistical reasoning is a deeply rich and complex process which is essential to comprehend in order to stay informed in civic matters and personal decision-making.

Strand	Statistical Reasoning - Measurement & Data		
Standard	**SR.C.1** Describe and compare measurable attributes.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations	**K.MD.A.1:** Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. **K.MD.A.2:** Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter*.	**1.MD.A.1:** Order three objects by length; compare the lengths of two objects indirectly by using a third object. **1.MD.A.2:** Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a*	**2.MD.A.4:** Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. **2.MD.A.1:** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. **2.MD.A.2:** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how

	K.MD.B.3: Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.)	*whole number of length units with no gaps or overlaps*.	the two measurements relate to the size of the unit chosen. **2.MD.A.3:** Estimate lengths using units of inches, feet, centimeters, and meters.
Strand	**Statistical Reasoning - Measurement & Data**		
Standard	**SR.C.2** Represent and interpret data.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations		**1.MD.C.4:** Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	**2.MD.D.9:** Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Organize and record data on a line plot, where the horizontal scale is marked off in whole-number units. **2.MD.D.10:** Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four

			categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.
Strand	**Statistical Reasoning - Measurement & Data**		
Standard	**SR.C.3** Relate addition and subtraction to length.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations			**2.MD.B.5:** Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. **2.MD.B.6:** Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.
Strand	**Statistical Reasoning - Measurement & Data**		
Standard	**SR.C.4** Work with time and money.		

	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations		**1.MD.B.3:** Tell and write time in hours and half-hours using analog and digital clocks. **1.MD.D.5:** Identify the coins and each corresponding value. (e.g. penny, nickel, dime, and quarter)	**2.MD.C.7:** Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. **2.MD.C.8:** Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?
Strand	**Statistical Reasoning - Measurement & Data**		
Standard	**SR.C.5** Solve problems involving measurement, conversion of measurement and estimation of intervals of time, liquid volumes, and masses of objects.		
	Childhood		
	Grade 3 Excludes compound units such as cm^3 and finding the geometric volume of a container Excludes multiplicative comparison problems (problems involving notions of "times as much")	Grade 4	Grade 5
Performance Expectations	**3.MD.A.1:** Tell and write time to the nearest minute and measure time intervals in minutes using analog and digital clocks. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the		

	problem on a number line diagram. **3.MD.A.2:** Measure and estimate liquid volumes and masses of objects using standard metric units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same metric units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.	**4.MD.A.1:** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …*	**5.MD.A.1:** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

		4.MD.A.2: Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	
Strand	**Statistical Reasoning - Measurement & Data**		
Standard	**SR.C.6** Represent and interpret data.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations	**3.MD.B.3:** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets*. **3.MD.B.4:** Generate measurement data by measuring lengths of	**4.MD.B.4:** Make a line plot to display a data set of	**5.MD.B.2:** Make a line plot to display a data set of

	objects using rulers marked with halves and fourths of an inch. Record and show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or fourths.	measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection*.	measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally*.
Strand	**Statistical Reasoning - Measurement & Data**		
Standard	**SR.C.7** Understand concepts of Geometric measurement: involving perimeter, area, and volume.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations	**3.MD.C.5:** Recognize area as an attribute of plane figures and understand concepts of area measurement. **3.MD.C.5a:** A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. **3.MD.C.5b:** A plane figure which can be covered without gaps or overlaps by		**5.MD.C.3:** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **5.MD.C.3a:** A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. **5.MD.C.3b:** A solid figure which can be packed without gaps or overlaps

	n unit squares is said to have an area of *n* square units. **3.MD.C.6:** Measure areas by counting unit squares (square cm, square m, square in, square ft, and non- standard units **3.MD.C.7:** Relate area to the operations of multiplication and addition. **3.MD.C.7a:** Find the area of a rectangle with whole-number side lengths by tiling it and show that the area is the same as would be found by multiplying the side lengths. **3.MD.C.7b:** Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems and represent whole-number products as rectangular		using *n* unit cubes is said to have a volume of *n* cubic units. **5.MD.C.4:** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and non-standard units. **5.MD.C.5:** Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. When finding volumes of objects answers will be in cubic units. **5.MD.C.5a:** Find the volume of a right rectangular prism with whole-number edge lengths by packing it with unit cubes and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to

	areas in mathematical reasoning. **3.MD.C.7c:** Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning. **3.MD.C.7d:** Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. **3.MD.D.8:** Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	**4.MD.A.3:** Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor*.	represent the associative property of multiplication. **5.MD.C.5b:** Apply the formulas *V = l × w × h* and *V = B × h (where B stands for the area of the base)* for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. **5.MD.C.5c:** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Strand	Statistical Reasoning - Measurement & Data		
Standard	**SR.C.8** Geometric measurement: understand concept of angle and measure angles.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations		**4.MD.C.5:** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: **4.Md.C.5a:** An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. **4.MD.C.5b:** An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. **4.MD.C.6:** Measure angles in whole-number degrees using a	

		protractor. Sketch angles of specified measure. **4.MD.C.7:** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	

Definitions:

Strand: A body of knowledge in a content area identified by a simple title.

Standard: Enduring understandings and skills that students can apply and transfer to contexts that are new to the student.

Performance Expectation: Building blocks to the standard and measurable articulations of what the student understands and can do.