Quantitative Reasoning

Quantitative reasoning is the application of basic mathematics skills to analyze and process real-world information.

In the **K-5 grades**, students use:

- numbers, including written numerals, to represent quantities and to solve quantitative problems.
- work on counting and cardinality, number and operations in Base Ten and fractions.
- develop strategies to extend their understanding of the base ten system and apply those strategies to solve real-world problems using all four operations.
- progress from working with whole numbers to fractions and decimals.

In **grades 6-8** students use:

- reasoning about multiplication and division to solve ratio and rate problems about quantities.
- develop an understanding of proportionality to solve problems and graph relationships.
- extend and develop their understanding of rational numbers and can compute in all operations.
- operations to solve real-world problems.
- understanding of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.
- reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

In **high school grades**, students use:

- the foundational concepts of operations with rational numbers and numerical properties built in the K-5 and 6-8 grade spans are applied to irrational numbers.
- a wider variety of units in modeling, (e.g. acceleration, currency conversions, and derived quantities such as person-hours and heating degree days), as well as the properties of rational and irrational numbers students are guided to the solution(s) to multi- step problems.
- Extend the properties of integer exponents to rational exponents deepens student understanding of how various but equivalent notations can facilitate their algebraic reasoning and problem-solving processes.
- expand these operations and properties into complex numbers, vectors, and matrices to further deepen their understanding of quantitative reasoning

Algebraic Reasoning

Algebraic reasoning is about generalizing arithmetic operations and determining unknown quantities by recognizing and analyzing patterns along with developing generalizations about these patterns.

In the **K-5 grades**, students:

- explore, analyze, represent, and generalize mathematical ideas and relationships.
- develop an understanding of the fundamental properties of number and operations
- understand the use of the equal sign to represent equivalence
- use quantitative reasoning to understand mathematical relationships.

Students in **grades 6-8** progress in their understanding of:

- variables in mathematical expressions and equations.
- expressions in different forms can be equivalent, use the properties of operations to rewrite expressions in equivalent forms, and describe relationships between quantities.
- begin to analyze and solve real-world and mathematical problems using equations and inequalities.
- construct and interpret tables and graphs.
- Understanding builds from writing and solving simple equations to solving proportional situations.
- exploring slope and y-intercept and relationships between variables, and eventually include multiple equations to solve systems of linear equations.
- the concept of a function is a rule that assigns one output to each input, and they learn to translate among different representations of functions.

In **high school grades**, students will continue to develop their understanding of:

- expressions, equations, functions and function notation.
- interpret the structure of algebraic expressions and be able to write expressions in equivalent forms to reveal information and to solve problems.
- perform arithmetic operations on polynomials and rewrite rational functions.
- the relationship between zeros and factors of polynomials will transition into using polynomial identities to solve problems.
- create equations that describe relationships and solve equations as a process of reasoning (with appropriate justification).
- represent and solve equations, inequalities, and systems of equations using a variety of mathematically sound techniques.
- interpret functions that arise in applications in terms of context and analyze functions using different representations.
- build functions that model relationships between two quantities, and build new functions from existing functions through transformations, combinations, compositions, and examining the inverse
- construct and compare linear, quadratic, and exponential models and use those models to solve problems.
- interpret expressions for functions in terms of the situation they model.
- extend their understanding of algebra and functions and apply similar processes of reasoning to polynomial, logarithmic and trigonometric functions and their graphs.

Geometric Reasoning

Geometric reasoning is the use of critical thinking, logical argument and spatial reasoning to solve problems and find new relationships. Students must first have a critical understanding of any underlying assumptions and relationships. This allows them to develop coherent knowledge and apply their reasoning skills.

In the **K-5 grades**, students will develop an understanding of:

- the attributes of two- and three-dimensional shapes and apply this knowledge to real-world problems.
- be introduced to the coordinate system.

Students in **grades 6-8** work with:

- two- and three-dimensional objects to reason about relationships among shapes.
- calculate area, surface area, volume, and circumference using multiple methods including decomposing shapes so that they can develop, justify, and use formulas including the Pythagorean Theorem and its converse.
- use scale drawings and informal constructions to gain familiarity with the relationships between angles formed by intersecting lines and transformations.

During **high school grades**, students begin to:

- formalize their geometry experiences from elementary and middle school, using more complex definitions and reasoning of proofs.
- make geometric constructions using a variety of technological tools and connect these explorations to reasoning and proofs.
- Attributes of parallel lines intersected by a transversal are further developed and extended into properties of triangles, quadrilaterals, and regular polygons as well as circles using informal and formal reasoning.
- Fundamental to the concepts of congruence, similarity, and symmetry are transformations which can preserve distance and angles.
- The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity.
- The Pythagorean Theorem along with these ratios are fundamental in many real-world and theoretical situations.
- Correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa.
- Concepts of two- and three-dimensional shapes are explored using algebraic formulas and modeling.
- extend their geometric reasoning through the exploration of trigonometric identities and properties of conic sections.

Statistical Reasoning

Statistical reasoning is the way people analyze data and make sense of information. It involves generalizations that connect one concept to another.

In the **K-5 grades**, students will develop:

- strategies to represent and interpret data,
- describe and compare measurable attributes,

- understand concepts of measurement including perimeter, area, volume, time, and money.

Students in **grades 6-8** continue to:

- develop their ability to think statistically.
- Measures of central tendency (mean, median, and mode) as well as measures of variability (range, interquartile range, mean absolute deviation) are used to describe data.
- Previous work with single data distributions is expanded to compare two data distributions and address questions about differences between populations.
- Informal work with random sampling and learning about the importance of representative samples for drawing inferences is introduced.
- expand their statistical understanding to include connections involving modeling with linear equations, as well as nonlinear expressions.
- Look for patterns in a bivariate data system is emphasized.

In **grades 9-12** students:

- extend their statistical understanding of univariate and bi-variate data in a real-world context.
- This understanding is used to make decisions or predictions based on the data.
- Since data can be variable, statistics provide the tools for taking this variability into account.
- Data can be categorical or quantitative in nature.
- Appropriate methods for collecting, displaying, summarizing, and analyzing data are learned and employed.
- Algebraic and geometric reasoning are utilized to create linear regression models in order to interpret the relationship between two quantitative variables when appropriate.
- The conditions under which data are collected and the use of randomization in the design of a study are necessary for drawing valid conclusions about the population under study.
    - Since random processes can be described mathematically by using a probability model, the role of probability in making predictions or in making decisions becomes evident.
    - Technology makes it possible to generate plots, find regression functions, compute correlation coefficients, and run simulations to better understand data.
    - Statistical reasoning is a deeply rich and complex process which is essential to comprehend in order to stay informed in civic matters and personal decision-making.

The Guiding Principles & Standards for Mathematical Practice

The Guiding Principles influence education in Maine and should be reflected throughout Mathematics curriculum. The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Full descriptions of the Guiding Principles and Standards for Mathematical Practice can be found in the Supplemental Material. Examples of how students can show evidence of those Guiding Principles and Standards for Mathematical Practice may include:

Guiding Principles

A. A clear and effective communicator: Students will use written, oral, symbolic, and visual forms of expression to communicate mathematically.
B. A self-directed and lifelong learner: Students generate and persevere in solving questions while demonstrating a growth mindset.
C. A creative and practical problem solver: Students will pose and solve mathematical problems by using a variety of strategies that connect to real-world examples.
D. A responsible and involved citizen: Students make sense of the world around them through mathematics including economic literacy.
E. An integrative and informed thinker: Students connect mathematics to other learning by understanding the interrelationships of mathematical ideas and the role math plays in other disciplines and life.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them: Students will plan strategies to use and persevere in solving math problems.
2. Reason abstractly and quantitatively: Students will think about numbers in many ways and make sense of numerical relationships as they solve problems.
3. Construct viable arguments and critique the reasoning of others: Students will explain their thinking and make sense of the thinking of others.
4. Model with mathematics: Students will use representations to show their thinking in a variety of ways.
5. Use appropriate tools strategically: Students will use math tools such as tables, diagrams, and technology to explore and deepen their understanding of concepts.
6. Attend to precision: Students will use precise mathematical language and check their work for accuracy.
7. Look for and make use of structure: Students will use their current mathematical understandings to identify patterns and structure to make sense of new learning.
8. Look for and express regularity in repeated reasoning: Students will look for patterns and rules to help create general methods and shortcuts that can be applied to similar mathematical problems.