Mathematical Representations

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"…is the goal of mathematics answer-getting or long-term mathematical understanding?"

-the Math Pact (2021)

Deep Mathematical Understanding

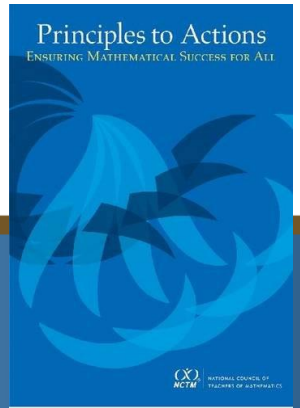
- Deep mathematical understanding comes from conceptual mathematical understanding partnered with procedural understanding
- Procedures cannot be taught in isolation to develop short-term or long-term mathematical understanding
- Teaching procedures and shortcuts disconnects students from developing deep mathematical understanding

Deep Mathematical Understanding

- Representations allow students to compare and contrast multiple depictions to help develop generalizations or patterns occurring in the mathematics
- Representations allow students to communicate their thinking about the mathematics
- Representations allow students to make sense of mathematical relationships
- Representations support deeper understanding of mathematical content compared to the use of abstract symbols and numbers alone

Eight Effective Teaching Practices

- NCTM's Principles to Actions: Ensuring Mathematical Success For All focuses on the Eight Effective Teaching Practices
  - These are eight practices that teachers do to support successful mathematical instruction (not to be confused with the eight Standards for Mathematical Practice from our state standards or the Common Core that focus on what students engage in within their mathematics learning)



Eight Effective Teaching Practices

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- **Use and connect mathematical representations.**
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.

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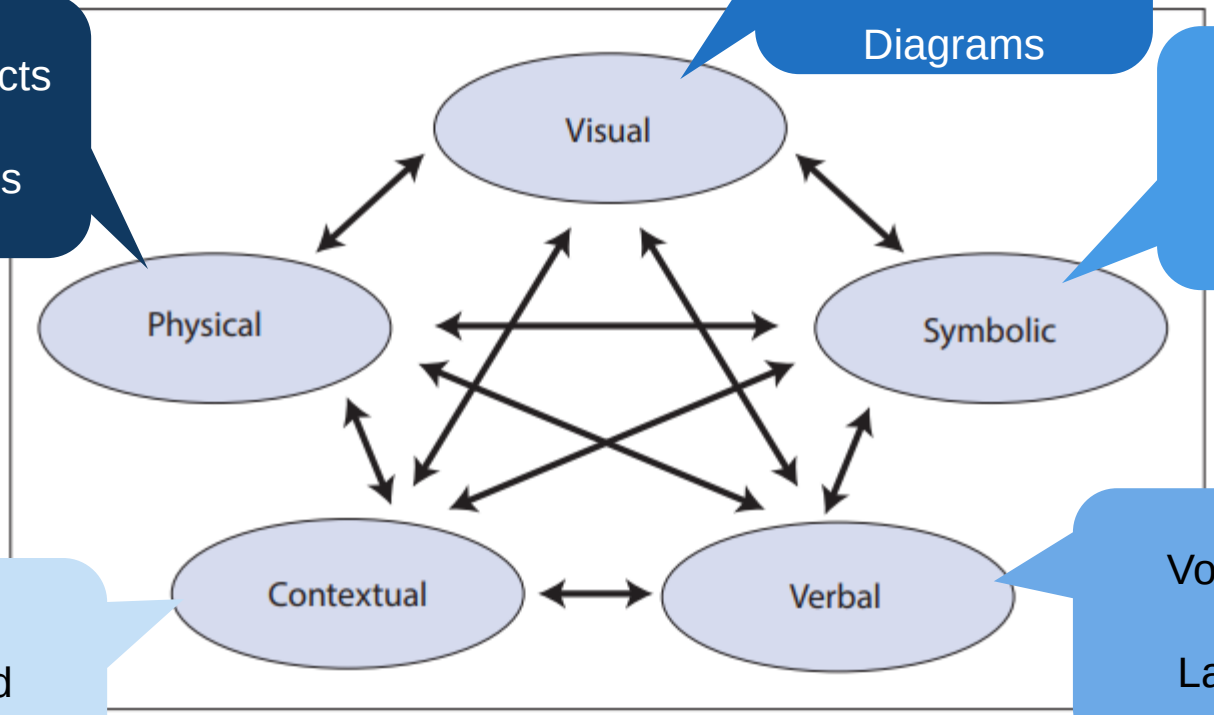
Use and connect mathematical representations.



Pictures

Models

Diagrams

Concrete objects

Manipulatives

Abstract

Numbers and Symbols

Vocabulary

Language

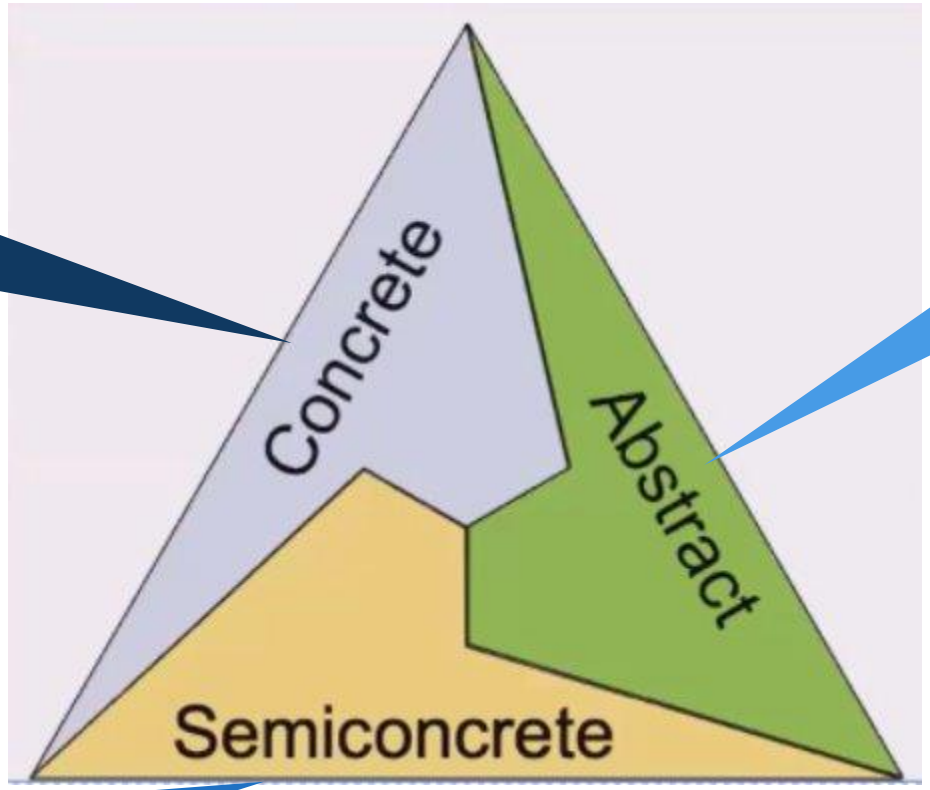
Real World Connections

NCTM's Principles to Actions, p 25

Types of Representations…Another Model

Physical objects

Manipulatives



Symbolic

Numbers and Symbols

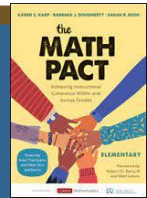
Visuals

Pictures

Models

Diagrams

the Math Pact – Elementary, p 52

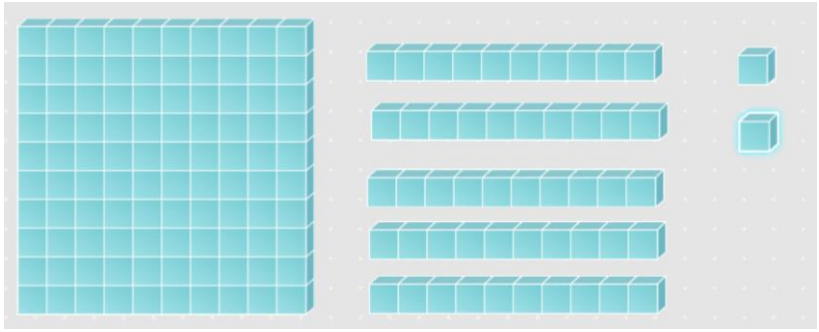


Examples of Concrete, SemiConcrete, and Abstract Representations

Concrete Representations	Semiconcrete Representations	Abstract Representations
Counters Base 10 materials Color tiles Rulers Number balance Three- and two-dimensional shapes Fraction pieces Decimal squares Percent necklace Two-color counters Cuisenaire Rods Connecting cubes Graphs made from physical materials	Drawings/sketches/ illustrations Schemas/graphic organizers Graphs Bar or strip diagram Place value mat Number lines Hundreds chart Grid paper Tables Virtual manipulatives Dynamic geometry software	Numbers Operation symbols Equal sign Inequality signs Words Tables Letters as unknowns Ordered pairs

the Math Pact – Elementary, p 58

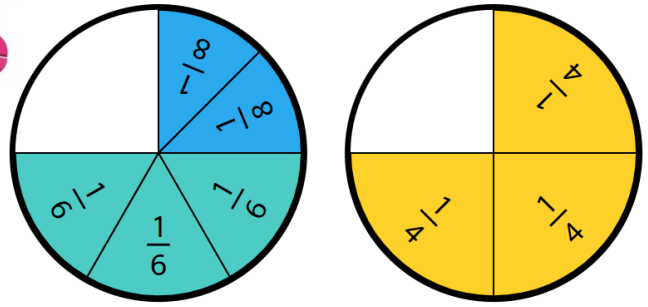
What does this look like?

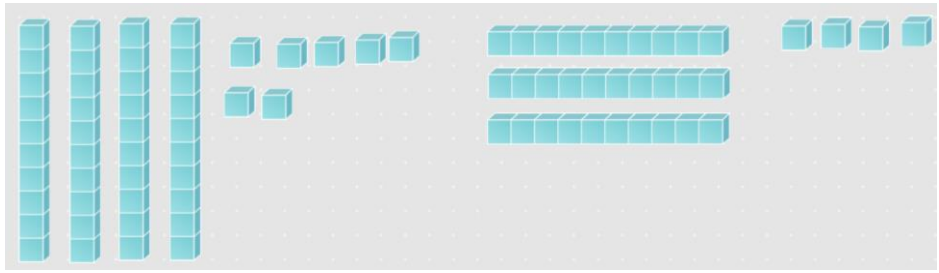


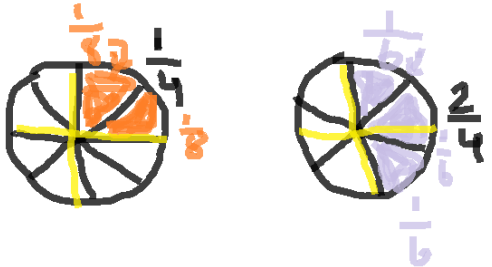
6

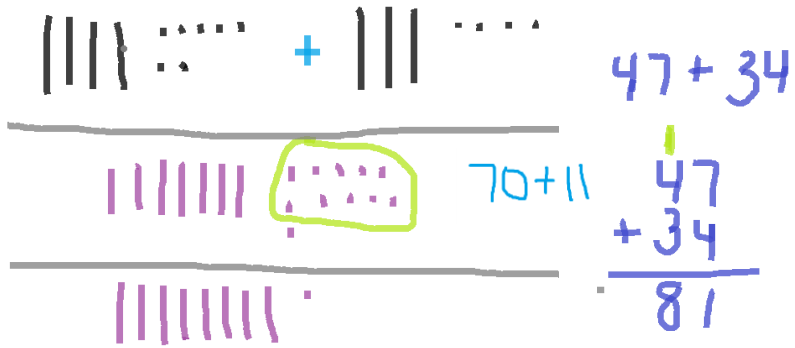
152

What does this look like?









$$\frac{2}{8} = \frac{1}{4} \qquad \frac{3}{6} = \frac{2}{4} \qquad \frac{2}{8} + \frac{3}{6} = \frac{3}{4}$$

$$47 + 34 = 81$$

Remember…

US Standard Algorithm Mastery is not expected until:

Addition → End of Grade 4

Subtraction → End of Grade 4

Multiplication → End of Grade 5

Division → End of Grade 6

Your Turn…

You are now a second-grade student. You have been working on addition and subtraction. In addition, you have used base 10 blocks, number lines, manipulatives, and expanded form to model addition. You have learned that sometimes you must regroup your ones or tens into tens or hundreds because you can only have one digit in each place value. So far in subtraction, you have subtracted with no regrouping, but you have used expanded form, number lines, base 10 blocks and other manipulatives.

Your Turn…

You are given the following two problems to solve. How can you use Concrete-Semiconcrete-Abstract representations to help support your learning?

Solve and show your thinking.

$72 - 29 = \_\_\_\_$

$$\begin{array}{r} 152 \\ -\ 86 \\ \hline \end{array}$$

https://toytheater.com/category/teacher-tools/

https://www.coolmath4kids.com/manipulatives/base-ten-blocks

Something to Think About…

ALL students can benefit from the CSA model or the **inclusion of manipulatives** and variety of **representations** to help **deepen mathematical understanding**. Manipulatives are not just for the youngest students. Please make them part of your **regular routines** and have them **accessible for all students**.

What are you going to change?

What is one change you can make this school year to help deepen mathematical understanding for your students?

Asynchronous Professional Learning

Mathematics Professional Learning

Welcome to the Mathematics Professional Learning page. Here you will find a variety of offerings from progressions of learning skills and operations, math manipulatives, best practices, diagnostic tools, and supports for all students. If you have ideas of other offering you would like to see please email us.

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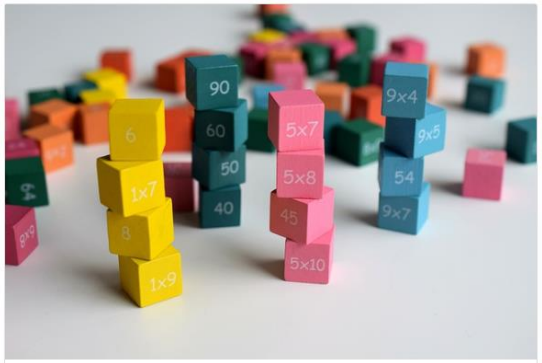
Progressions of Learning



Math Manipulatives

Math Practices

https://www.maine.gov/doe/pl/math

Any questions?

Maine Department of Education

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