Mathematics Standards Introduction

A strong mathematics education depends upon a clear understanding of its interrelated concepts, skills and practices to ensure students are on the pathway to success in their academic careers. The knowledge and skills students need to be prepared for mathematics in college, career, and life are woven throughout the K-12 mathematics performance expectations.

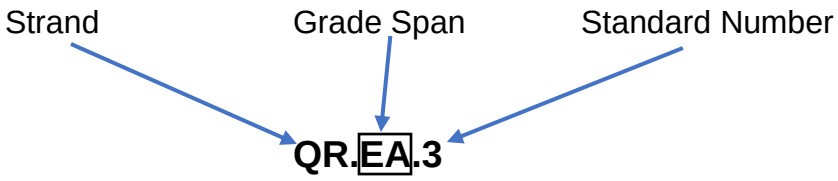
Outline of Mathematics Strands and Standards

These mathematical performance expectations are building blocks to standards. The standards are grouped into four strands:

- **Quantitative Reasoning** (Blue)**:** Counting and Cardinality, Number and Operations in Base Ten, Number and Operations Fractions, Ratio and Proportional Relations, The Number System, and Number and Quantity.
- **Algebraic Reasoning** (Green)**:** Operations and Algebraic Thinking, Expressions and Equations, Functions, and Algebra
- **Geometric Reasoning** (Red)**:** Geometry
- **Statistical Reasoning** (Purple)**:** Measurement and Data, Statistics and Probability

These mathematical performance expectations are broken into three grade spans: **C**hildhood (K-5), **E**arly **A**dolescence (6-8), and **A**dolescence (9-Diploma). The strands are color-coded, as indicated above, for continuity throughout the grade spans. Standards do not work in isolation, they are connected through and across strands.

How to Read the Standards



Within the high school performance expectations, modeling is woven throughout the four strands and is denoted with a star (★). The high school standards also contain some performance expectations which are denoted by a plus (**+**). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.

The Guiding Principles & Standards for Mathematical Practice

The Guiding Principles influence education in Maine and should be reflected throughout Mathematics curriculum. The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Full descriptions of the Guiding Principles and Standards for Mathematical Practice can be found in the Supplemental Material. Examples of how students can show evidence of those **Guiding Principles** and **Standards for Mathematical Practice** **may** include:

Guiding Principles

A. **A clear and effective communicator:** Students will use written, oral, symbolic, and visual forms of expression to communicate mathematically.

B. **A self-directed and lifelong learner:** Students generate and persevere in solving questions while demonstrating a growth mindset.

C. **A creative and practical problem solver:** Students will pose and solve mathematical problems by using a variety of strategies that connect to real-world examples.

D. **A responsible and involved citizen:** Students make sense of the world around them through mathematics including economic literacy.

E. **An integrative and informed thinker:** Students connect mathematics to other learning by understanding the interrelationships of mathematical ideas and the role math plays in other disciplines and life.

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them:** Students will plan strategies to use and persevere in solving math problems.

2. **Reason abstractly and quantitatively:** Students will think about numbers in many ways and make sense of numerical relationships as they solve problems.

3. **Construct viable arguments and critique the reasoning of others:** Students will explain their thinking and make sense of the thinking of others.

4. **Model with mathematics:** Students will use representations to show their thinking in a variety of ways.

5. **Use appropriate tools strategically:** Students will use math tools such as tables, diagrams, and technology to explore and deepen their understanding of concepts.

6. **Attend to precision:** Students will use precise mathematical language and check their work for accuracy.

7. **Look for and make use of structure:** Students will use their current mathematical understandings to identify patterns and structure to make sense of new learning.

8. **Look for and express regularity in repeated reasoning:** Students will look for patterns and rules to help create general methods and shortcuts that can be applied to similar mathematical problems.

Quantitative Reasoning

Quantitative reasoning is the application of basic mathematics skills to analyze and process real-world information. In the K-5 grades, students use numbers, including written numerals, to represent quantities and to solve quantitative problems. Students will work on counting and cardinality, number and operations in Base Ten and fractions. Students will develop strategies to extend their understanding of the base ten system and apply those strategies to solve real-world problems using all four operations. Students progress from working with whole numbers to fractions and decimals.

In grades 6-8 students use reasoning about multiplication and division to solve ratio and rate problems about quantities. They develop an understanding of proportionality to solve problems and graph relationships. Overall, students extend and develop their understanding of rational numbers and can compute in all operations. Students use these operations to solve real-world problems. Students use this understanding of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

In the high school grades, the foundational concepts of operations with rational numbers and numerical properties built in the K-5 and 6-8 grade spans are applied to irrational numbers. Using a wider variety of units in modeling, (e.g. acceleration, currency conversions, and derived quantities such as person-hours and heating degree days), as well as the properties of rational and irrational numbers students are guided to the solution(s) to multi-step problems. Extending the properties of integer exponents to rational exponents deepens student understanding of how various but equivalent notations can facilitate their algebraic reasoning and problem-solving processes. Students are encouraged to expand these operations and properties into complex numbers, vectors, and matrices to further deepen their understanding of quantitative reasoning.

Strand	**Quantitative Reasoning- Counting and Cardinality**		
Standard	**QR.C.1** Know the number names and the count sequence.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations	**K.CC.A.1:** Count to 100 by ones and by tens **K.CC.A.2:** Count forward beginning from a given number within the known sequence (instead of having to begin at 1) **K.CC.A.3:** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects)		
Strand	**Quantitative Reasoning- Counting and Cardinality**		
Standard	**QR.C.2** Count to tell the number of objects.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations	**K.CC.B.4:** Understand the relationship between numbers and quantities; connect counting to cardinality. **K.CC.B.4a:** When counting objects, say the number names in the standard order, pairing each object with one and only one number name		

	and each number name with one and only one object. **K.CC.B.4b:** Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. **K.CC.B.4c:** Understand that each successive number name refers to a quantity that is one larger. Recognize the one more pattern of counting using objects. **K.CC.B.5:** Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.		
Strand	**Quantitative Reasoning- Counting and Cardinality**		
Standard	**QR.C.3** Compare numbers.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations	**K.CC.C.6:** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another		

	group, e.g., by using matching and counting strategies. Include groups with up to ten objects. **K.CC.C.7:** Compare two numbers between 1 and 10 presented as written numerals.		
Strand	**Quantitative Reasoning - Numbers and Operations in Base Ten**		
Standard	**QR.C.4** Extend the counting sequence.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations		**1.NBT.A.1:** Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	
Strand	**Quantitative Reasoning - Numbers and Operations in Base Ten**		
Standard	**QR.C.5** Understand place value.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations	**K.NBT.A.1:** Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8 and 10+8=18); understand that these numbers are composed of	**1.NBT.B.2:** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: **1.NBT.B.2a:** 10 can be thought of as a bundle of ten ones - called a “ten.”	**2.NBT.A.1:** Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: **2.NBT.A.1a:** 100 can be thought of as a bundle of ten tens - called a “hundred.”

	ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	**1.NBT.B.2b:** The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. **1.NBT.B.2c:** The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). **1.NBT.B.3:** Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	**2.NBT.A.1b:** The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). **2.NBT.A.2:** Count within 1000; skip-count by 5s, 10s, and 100s. Identify patterns in skip counting at any number. (For example, 37, 47, 57 or 328, 428, 528, etc.) **2.NBT.A.3:** Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. **2.NBT.A.4:** Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
Strand	**Quantitative Reasoning - Numbers and Operations in Base Ten**		
Standard	**QR.C.6** Use place value understanding and properties of operations to add and subtract.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations			**2.NBT.B.5:** Fluently add and subtract within 100 using strategies based on place value, properties of operations,

			and/or the relationship between addition and subtraction.
		1.NBT.C.4: Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	**2.NBT.B.7:** Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
		1.NBT.C.5: Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	**2.NBT.B.8:** Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. **2.NBT.B.6:** Add up to four two-digit numbers using strategies based on place value and properties of operations. **2.NBT.B.9:** Explain why addition and subtraction strategies work, using place value and the properties of operations.

		1.NBT.C.6: Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	Explanations may be supported by drawings or objects.
Strand	**Quantitative Reasoning - Numbers and Operations in Base Ten**		
Standard	**QR.C.7** Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers and decimals to hundredths.		
	Childhood		
	Grade 3 A range of algorithms may be used.	Grade 4 Grade 4 expectations in this *strand* are limited to whole numbers less than or equal to 1,000,000	Grade 5
Performance Expectations	**3.NBT.A.2:** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. **3.NBT.A.3:** Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.	**4.NBT.B.4:** Fluently add and subtract multi-digit whole numbers using the standard algorithm. **4.NBT.B.5:** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place	**5.NBT.B.5:** Fluently multiply multi-digit whole numbers using the standard algorithm.

		value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **4.NBT.B.6:** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	**5.NBT.B.6:** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **5.NBT.B.7:** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, money and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
Strand	**Quantitative Reasoning - Numbers and Operations in Base Ten**		
Standard	**QR.C.8** Understand the place value system.		
	Childhood		
	Grade 3 A range of algorithms may be used.	Grade 4 Grade 4 expectations in this *strand* are limited to whole numbers less than or equal to 1,000,000	Grade 5

Performance Expectations	**3.NBT.A.1:** Use place value understanding to round whole numbers to the nearest 10 or 100.	**4.NBT.A.3:** Use place value understanding to round multi-digit whole numbers to any place. **4.NBT.A.2:** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **4.NBT.A.1:** Recognize that in a multi-digit whole number, a digit in any place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division*.	**5.NBT.A.4:** Use place value understanding to round decimals to any place. **5.NBT.A.3:** Read, write, and compare decimals to thousandths. **5.NBT.A.3a:** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). **5.NBT.A.3b:** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **5.NBT.A.1:** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. **5.NBT.A.2:** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-

			number exponents to denote powers of 10.
Strand	**Quantitative Reasoning - Numbers and Operations: Fractions**		
Standard	**QR.C.9** Develop and extend the understanding of fractions as numbers, including equivalence and ordering.		
	Childhood		
	Grade 3 Grade 3 expectations in this *strand* are limited to fractions with denominators 2, 3, 4, 6, and 8.	Grade 4 Grade 4 expectations in this *strand* are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	Grade 5
Performance Expectations	**3.NF.A.1:** Understand a unit fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*. **3.NF.A.2:** Understand a fraction as a number on the number line; represent fractions on a number line diagram. **3.NF.A.2a:** Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.		

	3.NF.A.2b: Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line. **3.NF.A.3:** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **3.NF.A.3a:** Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. **3.NF.A.3b:** Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model. **3.NF.A.3c:** Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram*.	**4.NF.A.1:** Explain why a fraction *a*/*b* is equivalent to a fraction ($n \times a$)/($n \times b$) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions, including fractions greater than 1.	

	3.NF.A.3d: Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	**4.NF.A.2:** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	
Strand	**Quantitative Reasoning - Numbers and Operations: Fractions**		
Standard	**QR.C.10** Understand decimal notation for fractions, and compare decimal fractions.		
	Childhood		
	Grade 3	Grade 4 Grade 4 expectations in this *strand* are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.	Grade 5
Performance Expectations		**4.NF.C.5:** Express a fraction with denominator 10 as an equivalent fraction with denominator 100 and use this technique to add two fractions with respective denominators 10 and 100.[2] *For*	

		example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. **4.NF.C.6:** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram*. **4.NF.C.7:** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	
Strand	**Quantitative Reasoning - Numbers and Operations: Fractions**		
Standard	**QR.C.11** Use equivalent fractions as a strategy to add and subtract fractions.		
	Childhood		
	Grade 3	Grade 4 Grade 4 expectations in this *strand* are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.	Grade 5

Performance Expectations		**4.NF.B.3:** Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. **4.NF.B.3a:** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. **4.NF.B.3b:** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model to build fractions from unit fractions. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8*. **4.NF.B.3c:** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. **4.NF.B.3d:** Solve word problems involving addition and	**5.NF.A.1:** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* **5.NF.A.2:** Solve word problems involving addition and subtraction of fractions

		subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2*.
Strand	**Quantitative Reasoning - Numbers and Operations: Fractions**		
Standard	**QR.C.12** Apply and extend previous understandings of multiplication and division to multiply and divide fractions.		
	Childhood		
	Grade 3	Grade 4	Grade 5 Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.
Performance Expectations		**4.NF.B.4:** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. **4.NF.B.4a:** Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4)*. **4.NF.B.4b:** Understand a multiple of a/b as a multiple of 1/b and use this understanding to multiply a	**5.NF.B.4:** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **5.NF.B.4a:** Interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = (ac)/(bd).*

		fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* **4.NF.B.4c:** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*	**5.NF.B.4b:** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas. **5.NF.B.3:** Interpret a fraction as division of the numerator by the denominator (*a*/*b* = *a* ÷ *b*). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* **5.NF.B.5:** Interpret multiplication scaling (resizing), by: **5.NF.B.5a:** Comparing the size of a product to the size of one factor on

			the basis of the size of the other factor, without performing the indicated multiplication. **5.NF.B.5b:** Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. **5.NF.B.6:** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **5.NF.B.7:** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1] **5.NF.B.7a:** Interpret division of a unit fraction by a non-zero whole number and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the*

			relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3. **5.NF.B.7b:** Interpret division of a whole number by a unit fraction and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4*. **5.NF.B.7c:** Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Strand	**Quantitative Reasoning- Ratio and Proportional Relationships**
Standard	**QR.EA.1** Understand ratio and rate concepts and use ratio and rate reasoning to solve problems.
	Early Adolescence
	Grades 6-8
Performance Expectations	**6.RP.A.1:** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks of the chickadees in the pine tree was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

	6.RP.A.2: Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0 and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 5 lobsters, which is a rate of $15 per lobster."* *Expectations for unit rates in this grade are limited to non-complex fractions.* **6.RP.A.3:** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **6.RP.A.3a:** Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. **6.RP.A.3b:** Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* **6.RP.A.3c:** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. **6.RP.A.3d:** Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
Strand	**Quantitative Reasoning- Ratio and Proportional Relationships**
Standard	**QR.EA.2** Analyze proportional relationships and use them to solve real-world and mathematical problems.
	Early Adolescence
	Grades 6-8
Performance Expectations	**7.RP.A.1:** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $\frac{\frac{1}{2}}{\frac{1}{4}}$ *miles per hour, equivalently 2 miles per hour*. **7.RP.A.2:** Recognize and represent proportional relationships between quantities. **7.RP.A.2a:** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

	7.RP.A.2b: Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. **7.RP.A.2c:** Represent proportional relationships by equations. *For example, if the total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn*. **7.RP.A.2d:** Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate. **7.RP.A.3:** Use proportional relationships to solve multistep ratio, rate, and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*
Strand	**Quantitative Reasoning- The Number System**
Standard	**QR.EA.3** Apply and extend previous understandings of operations with whole numbers to rational numbers.
	Early Adolescence
	Grades 6-8
Performance Expectations	**6.NS.A.1:** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and/or equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.)* **6.NS.B.3:** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. **7.NS.A.1:** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. **7.NS.A.1a:** Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has a zero charge because its two constituents are oppositely charged.* **7.NS.A.1b:** Understand *p* + *q* as the number located a distance |*q*| from *p*, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

	7.NS.A.1c: Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts. **7.NS.A.1d:** Apply properties of operations as strategies to add and subtract rational numbers. **7.NS.A.2:** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. **7.NS.A.2a:** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. **7.NS.A.2b:** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. **7.NS.A.2c:** Apply properties of operations as strategies to multiply and divide rational numbers. **7.NS.A.2d:** Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **7.NS.A.3:** Solve real-world and mathematical problems involving the four operations with rational numbers. *Computations with rational numbers extend the rules for manipulating fractions to complex fractions.*
Strand	**Quantitative Reasoning- The Number System**
Standard	**QR.EA.4** Compute fluently with multi-digit whole numbers and find common factors and multiples.
	Early Adolescence
	Grades 6-8
Performance Expectations	**6.NS.B.2:** Fluently divide multi-digit numbers using the standard algorithm. **6.NS.B.4:** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. (*For example: Use prime factorization to find the greatest common factor)*; Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two relatively prime numbers. *For example, express 36 + 8 as 4 (9 + 2).*
Strand	**Quantitative Reasoning - The Number System**

Standard	**QR.EA.5** Apply and extend previous understandings of numbers to the system of rational numbers.
	Early Adolescence
	Grades 6-8
Performance Expectations	**6.NS.C.5:** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative rational numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **6.NS.C.6:** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **6.NS.C.6a:** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite. **6.NS.C.6b:** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. **6.NS.C.6c:** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **6.NS.C.7:** Understand ordering and absolute value of rational numbers. **6.NS.C.7a:** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right*. **6.NS.C.7b:** Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write -3°C > -7°C to express the fact that -3°C is warmer than -7°C*. **6.NS.C.7c:** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars*. **6.NS.C.7d:** Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars*.

	6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
Strand	**Quantitative Reasoning- The Number System**
Standard	**QR.EA.6** Know that there are numbers that are not rational, and approximate them by rational numbers.
	Early Adolescence
	Grades 6-8
Performance Expectations	**8.NS.A.1:** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansions terminate in 0s or eventually repeats and convert a decimal expansion into a rational number. **8.NS.A.2:** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations*.

Strand	**Quantitative Reasoning - Number and Quantity: The Real Number System**
Standard	**QR.A.1** Extend the properties of exponents to rational exponents.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSN.RN.A.1:** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5*. **HSN.RN.A.2:** Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Strand	**Quantitative Reasoning- Number and Quantity: The Real Number System**

Standard	**QR.A.2** Use properties of rational and irrational numbers.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSN.RN.B.3:** Explain when and why the sum or product of two rational and/or irrational numbers is rational or irrational.
Strand	**Quantitative Reasoning - Number and Quantity: Quantities** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).**
Standard	**QR.A.3** Reason quantitatively and use units to solve problems. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSN.Q.A.1:** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. Example: Marlena made a scale drawing of the sand volleyball court at her summer camp. The drawing of the volleyball court is 6 cm long by 3 cm wide. The actual volleyball court is 18 meters long. What scale did Marlena use for the drawing? ★ **HSN.Q.A.2:** Define appropriate quantities for the purpose of descriptive modeling. Example: If a town in Aroostook county with a population of 1254 people is projected to double in size every 105 years, what will the population be 315 years from now? ★ **HSN.Q.A.3:** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. Example: The label on a ½ - liter bottle of flavored water bottled in Maine indicates that one serving of 8 ounce contains 60 calories. The label also says that the full bottle contains 130 calories. Is this the actual amount or the estimated amount of calories in this bottle? How would you explain any discrepancy? ★

Strand	**Quantitative Reasoning - Number and Quantity: Complex Number System** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**QR.A.4 (+)** Perform arithmetic operations with complex numbers.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**(+) HSN.CN.A.1:** Know there is a complex number *i* (which is a non-real number) such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real. **(+) HSN.CN.A.2:** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **(+) HSN.CN.A.3:** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
Strand	**Quantitative Reasoning - Number and Quantity: Complex Number System** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**QR.A.5 (+)** Represent complex numbers and their operations on the complex plane.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**(+) HSN.CN.B.4:** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **(+) HSN.CN.B.5:** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\, i)^3 = 8$ *because* $(-1 + \sqrt{3}\, i)$ *has modulus 2 and argument 120°.*

	(+) HSN.CN.B.6: Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
Strand	**Quantitative Reasoning - Number and Quantity: Complex Number System** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**QR.A.6 (+)** Use complex numbers in polynomial identities and equations.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**(+) HSN.CN.C.7:** Solve quadratic equations with real coefficients that have complex solutions. **(+) HSN.CN.C.8:** Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$. **(+) HSN.CN.C.9:** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
Strand	**Quantitative Reasoning - Number and Quantity: Vector and Matrix Quantities** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**QR.A.7 (+)** Represent and model with vector quantities.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**(+) HSN.VM.A.1:** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $\lvert\boldsymbol{v}\rvert$, $\lVert\boldsymbol{v}\rVert$, v). **(+) HSN.VM.A.2:** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. **(+) HSN.VM.A.3:** Solve problems involving velocity and other quantities that can be represented by vectors.

Strand	**Quantitative Reasoning - Number and Quantity: Vector and Matrix Quantities** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**QR.A.8 (+)** Perform operations on vectors.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**(+) HSN.VM.B.4:** Add and subtract vectors. **(+) HSN.VM.B4a:** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. **(+) HSN.VM.B4b:** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. **(+) HSN.VM.B4c:** Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. **(+) HSN.VM.B.5:** Multiply a vector by a scalar. **(+) HSN.VM.B5a:** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. **(+) HSN.VM.B5b:** Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\|c\boldsymbol{v}\| = |c|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).
Strand	**Quantitative Reasoning - Number and Quantity: Vector and Matrix Quantities** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**QR.A.9 (+)** Perform operations on matrices and use matrices in applications.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**(+) HSN.VM.C.6:** Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

	(+) HSN.VM.C.7: Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. **(+) HSN.VM.C.8:** Add, subtract, and multiply matrices of appropriate dimensions. **(+) HSN.VM.C.9:** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. **(+) HSN.VM.C.10:** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. **(+) HSN.VM.C.11:** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. **(+) HSN.VM.C12:** Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Algebraic Reasoning

Algebraic reasoning is about generalizing arithmetic operations and determining unknown quantities by recognizing and analyzing patterns along with developing generalizations about these patterns. In this K-5 strand, students explore, analyze, represent, and generalize mathematical ideas and relationships. Students will develop an understanding of the fundamental properties of number and operations, understand the use of the equal sign to represent equivalence, and use quantitative reasoning to understand mathematical relationships.

Students in grades 6-8 progress in their understanding of variables in mathematical expressions and equations. They understand that expressions in different forms can be equivalent, use the properties of operations to rewrite expressions in equivalent forms, and describe relationships between quantities. Students begin to analyze and solve real-world and mathematical problems using equations and inequalities. They construct and interpret tables and graphs. Understanding builds from writing and solving simple equations to solving proportional situations. These skills lead to exploring slope and y-intercept and relationships between variables, and eventually include multiple equations to solve systems of linear equations. Students grow to understand that the concept of a function is a rule that assigns one output to each input, and they learn to translate among different representations of functions.

In grades 9-12, students will continue to develop their understanding of expressions, equations, functions and function notation. They will interpret the structure of algebraic expressions and be able to write expressions in equivalent forms to reveal information and to solve problems. Students will perform arithmetic operations on polynomials and rewrite rational functions. An understanding of the relationship between zeros and factors of polynomials will transition into using polynomial identities to solve problems. Students will create equations that describe relationships and solve equations as a process of reasoning (with appropriate justification). They will represent and solve equations, inequalities, and systems of equations using a variety of mathematically sound techniques.

Students will interpret functions that arise in applications in terms of context and analyze functions using different representations. They will build functions that model relationships between two quantities, and build new functions from existing functions through transformations, combinations, compositions, and examining the inverse. Students will construct and compare linear, quadratic, and exponential models and use those models to solve problems. They will interpret expressions for functions in terms of the situation they model. Students will be encouraged to extend their understanding of algebra and functions and apply similar processes of reasoning to polynomial, logarithmic and trigonometric functions and their graphs.

Strand	**Algebraic Reasoning - Operations and Algebraic Thinking**		
Standard	**AR.C.1** Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.		
	Childhood		
	Kindergarten (Drawings need not show detail but should show the mathematics in the problem. This applies wherever drawings are mentioned in the Standards.)	Grade 1	Grade 2
Performance Expectations	**K.OA.A.1:** Represent addition and subtraction with objects, fingers, mental images, drawings sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. **K.OA.A.2:** Solve addition and subtraction word problems, and add and subtract within 10, (e.g., by using objects or drawings to represent the problem).	**1.OA.A.1:** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	**2.OA.A.1:** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

	K.OA.A.3: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). **K.OA.A.4:** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. **K.OA.A.5:** Fluently add and subtract within 5 including zero.	**1.OA.A.2:** Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.)	
Strand	**Algebraic Reasoning - Operations and Algebraic Thinking**		

Standard	**AR.C.2** Understand and apply properties of operation and the relationship between addition and subtraction within 20.		
	Childhood		
	Kindergarten	Grade 1 Students need not use formal terms for these properties.	Grade 2
Performance Expectations		**1.OA.B.3:** Apply properties of operations as strategies to add. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.) a + 0 = a (Additive identity property of 0)* **1.OA.B.4:** Understand subtraction as an unknown-addend problem. *For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.* **1.OA.C.5:** Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). **1.OA.C.6:** Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.	**2.OA.B.2:** Fluently add and subtract within 20 using mental strategies. By end of Grade 2,

		Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). **1.OA.D.7:** Understand the meaning of the equal sign and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2. **1.OA.D.8:** Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ - 3, 6 + 6 = _.*	know from memory all sums of two one-digit numbers.
Strand	**Algebraic Reasoning - Operations and Algebraic Thinking**		

Standard	**AR.C.3** Work with equal groups of objects to gain foundations for multiplication.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations			**2.OA.C.3:** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. **2.OA.C.4:** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
Strand	**Algebraic Reasoning - Operations and Algebraic Thinking**		
Standard	**AR.C.4** Understand properties of multiplication and the relationship between multiplication and division to represent and solve problems within 100.		
	Childhood		
	Grade 3 Students need not use formal terms for these properties.	Grade 4	Grade 5
Performance Expectations	**3.OA.A.1:** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5		

	groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7*. **3.OA.A.2:** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8*. **3.OA.A.3:** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.		

	3.OA.A.4: Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?* **3.OA.B.5:** Apply properties of operations as strategies to multiply. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* **3.OA.B.6:** Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the*		

	number that makes 32 when multiplied by 8. **3.OA.C.7:** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.		
Strand	**Algebraic Reasoning - Operations and Algebraic Thinking**		
Standard	**AR.C.5** Solve problems involving the four operations.		
	Childhood		
	Grade 3 This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in conventional order when there are no parentheses to specify a particular order (Order of Operations).	Grade 4	Grade 5
Performance Expectations		**4.OA.A.1:** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	

	3.OA.D.8: Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	**4.OA.A.2:** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. **4.OA.A.3:** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	
Strand	**Algebraic Reasoning - Operations and Algebraic Thinking**		
Standard	**AR.C.6** Gain familiarity with factors and multiples.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations		**4.OA.B.4:** Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is	

		a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	
Strand	**Algebraic Reasoning - Operations and Algebraic Thinking**		
Standard	**AR.C.7** Write and interpret numerical expressions.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations			**5.OA.A.1:** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. **5.OA.A.2:** Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*
Strand	**Algebraic Reasoning - Operations and Algebraic Thinking**		
Standard	**AR.C.8** Identify, explain, generate and analyze patterns.		

	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations	**3.OA.D.9:** Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends*.	**4.OA.C.5:** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way*.	**5.OA.B.3:** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so*.

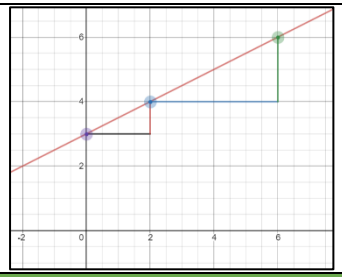
Strand	**Algebraic Reasoning - Expressions and Equations**
Standard	**AR.EA.1** Apply and extend previous understandings of arithmetic to algebraic expressions.
	Early Adolescence
	Grades 6-8
Performance Expectations	**6.EE.A.1:** Write and evaluate numerical expressions involving whole-number exponents. **6.EE.A.2:** Write, read, and evaluate expressions in which letters represent numbers. **6.EE.A.2a:** Write expressions that record operations with numbers and with letters representing numbers. *For example, express the calculation "Subtract y from 5" as 5 - y*.

	6.EE.A.2b: Identify parts of an expression using mathematical terms (including but not limited to: sum, term, product, factor, quotient, coefficient, variable, constant); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (x + 7) as a product of two factors; view (x + 7) as both a single entity and a sum of two terms*. **6.EE.A.2c:** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, using the order of operations. *For example, use the formulas $V = s^3$ and $A = 6\ s^2$ to find the volume and surface area of a cube with sides of length s = 1/2*. **6.EE.A.3:** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to factor the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y*. **6.EE.A.4:** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*
Strand	**Algebraic Reasoning - Expressions and Equations**
Standard	**AR.EA.2** Reason about and solve one-variable equations and inequalities.
	Early Adolescence
	Grades 6-8
Performance Expectations	**6.EE.B.5:** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **6.EE.B.6:** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.EE.B.7:** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. *For example, Sal is paid $0.50*

	per pound of blueberries that she rakes. If she rakes x pounds, and earns $17.25, write and solve an equation that determines how many pounds she raked. **6.EE.B.8:** Write an inequality of the form *x* > *c* or *x* < *c* to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form *x* > *c* or *x* < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
Strand	**Algebraic Reasoning - Expressions and Equations**
Standard	**AR.EA.3** Represent and analyze quantitative relationships between dependent and independent variables.
	Early Adolescence
	Grades 6-8
Performance Expectations	**6.EE.C.9:** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*
Strand	**Algebraic Reasoning - Expressions and Equations**
Standard	**AR.EA.4** Use properties of operations to generate equivalent expressions.
	Early Adolescence
	Grades 6-8
Performance Expectations	**7.EE.A.1:** Apply properties of operations to add, subtract, factor, and expand linear expressions with rational coefficients. *For example, 4x + 2 = 2(2x+1) and -3(x-5/3) = -3x +5* **7.EE.A.2:** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, A shirt is on sale for 20% off the regular price, p. The discount can be expressed as 0.2p. The new price for the shirt can be expressed as p – 0.2p or 0.8p.*
Strand	**Algebraic Reasoning - Expressions and Equations**

Standard	**AR.EA.5** Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
	Early Adolescence
	Grades 6-8
Performance Expectations	**7.EE.B.3:** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation*. **7.EE.B.4:** Use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities. **7.EE.B.4a:** Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* **7.EE.B.4b:** Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make and describe the solutions*.
Strand	**Algebraic Reasoning - Expressions and Equations**
Standard	**AR.EA.6** Work with radicals and integer exponents.
	Early Adolescence
	Grades 6-8
Performance Expectations	**8.EE.A.1:** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = (1/3)^3 = 1/27$.*

	8.EE.A.2: Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. **8.EE.A.3:** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger*. **8.EE.A.4:** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
Strand	**Algebraic Reasoning - Expressions and Equations**
Standard	**AR.EA.7** Understand the connections between proportional relationships, lines, and linear equations.
	Early Adolescence
	Grades 6-8
Performance Expectations	**8.EE.B.5:** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* **8.EE.B.6:** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. *For example, given the line $y = 0.5x + 3$ explain why the similar triangles have the same slope.*

	
Strand	**Algebraic Reasoning - Expressions and Equations**
Standard	**AR.EA.8** Analyze and solve linear equations and pairs of simultaneous linear equations.
	Early Adolescence
	Grades 6-8
Performance Expectations	**8.EE.C.7:** Solve linear equations in one variable. **8.EE.C.7a:** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). **8.EE.C.7b:** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. **8.EE.C.8:** Analyze and solve pairs of simultaneous linear equations. **8.EE.C.8a:** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. **8.EE.C.8b:** Solve systems of two linear equations in two variables algebraically (i.e. by substitution or elimination) and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6*. **8.EE.C.8c:** Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair*.
Strand	**Algebraic Reasoning - Functions**

Standard	**AR.EA.9** Define, evaluate, and compare functions in order to model relationships between quantities.
	Early Adolescence
	Grades 6-8 Function notation is not required for Grade 8.
Performance Expectations	**8.F.A.1:** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. **8.F.A.2:** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change*. **8.F.A.3:** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line*. **8.F.B.4:** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **8.F.B.5:** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Strand	**Algebraic Reasoning - Algebra: Seeing Structure in Expressions** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).**

Standard	**AR.A.1** Interpret the structure of expressions.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSA.SSE.A.1:** Interpret expressions that represent a quantity in terms of its context. ★ **SSE.A.1a:** Interpret parts of an expression, such as terms, factors, and coefficients. ★ **SSE.A.1b:** Interpret multi-part expressions by viewing one or more of their parts as a single entity. *For example, view $P(1+r)^n$ as the product of P and a factor not depending on P and interpret the parts.* ★ **HSA.SSE.A.2:** Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, allowing for it to be recognized as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*
Strand	**Algebraic Reasoning - Algebra: Seeing Structure in Expressions** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).**
Standard	**AR.A.2** Write expressions in equivalent forms to reveal information and to solve problems. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSA.SSE.B.3:** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★ **HSA.SSE.B.3a:** Rewrite a quadratic expression (such as by factoring) to reveal the zeros of the function it defines. ★ **HSA.SSE.B.3b:** Rewrite a quadratic expression (such as by completing the square) to reveal the maximum or minimum value of the function it defines. ★ **HSA.SSE.B.3c:** Use the properties of exponents to transform expressions for exponential functions. *For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* ★ **HSA.SSE.B.4:** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, Watermilfoil in one Maine lake triples in the number of plants each week during the summer when boat propellers are not cleared when exiting the lake. If the lake has 20*

	plants at the beginning of the season, how many plants will exist at the end of the 12-week summer season? What is the general formula for Watermilfoil growth for this lake? ★
Strand	**Algebraic Reasoning - Algebra: Arithmetic with Polynomials & Rational Expressions**
Standard	**AR.A.3** Perform arithmetic operations on polynomials.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSA.APR.A.1:** Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. **HSA.APR.A.1a**: Perform operations on polynomial expressions (addition, subtraction, multiplication, and division), and compare the system of polynomials to the system of integers. **HSA.APR.A.1b**: Factor and/or expand polynomial expressions, identify and combine like terms, and apply the Distributive Property.
Strand	**Algebraic Reasoning - Algebra: Arithmetic with Polynomials & Rational Expressions** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.4** Understand the relationship between zeros and factors of polynomials.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSA.APR.B.2:** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. For example, consider the polynomial function $P(x) = x^4 – 2x^3 + ax^2 + 8x + 12$, where a is an unknown real number. If (x-3) is a factor of this polynomial, what is the value of a? **(+) HSA.APR.B.3:** Identify zeros of polynomials of degree three or higher when suitable factorizations (in factored form or easily factorable) are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Strand	**Algebraic Reasoning - Algebra: Arithmetic with Polynomials & Rational Expressions**

	The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.
Standard	**AR.A.5 (+)** Use polynomial identities to solve problems.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**(+) HSA.APR.C.4:** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.* **(+) HSA.APR.C.5:** Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.
Strand	**Algebraic Reasoning - Algebra: Arithmetic with Polynomials & Rational Expressions** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.6** Rewrite rational expressions.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSA.APR.D.6:** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. **(+) HSA.APR.D.7:** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
Strand	**Algebraic Reasoning -Algebra: Creating Equations and/or Inequalities** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).**
Standard	**AR.A.7** Create equations and/or inequalities that describe numbers or relationships. ★

	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSA.CED.A.1:** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*. ★ **HSA.CED.A.2:** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★ **HSA.CED.A.3:** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods such as lobsters, blueberries, and potatoes.* ★ **HSA.CED.A.4:** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R*. ★
Strand	**Algebraic Reasoning - Algebra: Reasoning with Equations & Inequalities**
Standard	**AR.A.8** Understand solving equations as a process of reasoning and explain the reasoning.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSA.REI.A.1:** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method. **HSA.REI.A.2:** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
Strand	**Algebraic Reasoning - Algebra: Reasoning with Equations & Inequalities** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.9** Solve equations and inequalities in one variable.
	Adolescence
	Grades 9-Diploma

Performance Expectations	**HSA.REI.B.3:** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **HSA.REI.B.4:** Solve quadratic equations in one variable. **HSA.REI.B.4a:** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. **HSA.REI.B.4b: i)** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. **(+) HSA.REI.B.4b: ii)** Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
Strand	**Algebraic Reasoning - Algebra: Reasoning with Equations & Inequalities** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.10** Solve systems of equations.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSA.REI.C.5:** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. **HSA.REI.C.6:** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **HSA.REI.C.7:** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the point(s) of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.* **(+) HSA.REI.C.8:** Represent a system of linear equations as a single matrix equation in a vector variable. **(+) HSA.REI.C.9:** Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).
Strand	**Algebraic Reasoning - Algebra: Reasoning with Equations & Inequalities**

	Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).
Standard	**AR.A.11** Represent and solve equations and inequalities graphically.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSA.REI.D.10:** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation. **HSA.REI.D.11:** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★ **HSA.REI.D.12:** Graph the solutions of a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set of a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
Strand	**Algebraic Reasoning - Functions: Interpreting Functions**
Standard	**AR.A.12** Understand the concept of a function and use function notation.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSF.IF.A.1:** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$. **HSF.IF.A.2:** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **HSF.IF.A.3:** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1*.

Strand	**Algebraic Reasoning - Functions: Interpreting Functions** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).**
Standard	**AR.A.13** Interpret functions that arise in applications in terms of the context. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSF.IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features may include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative and absolute maximums and minimums; symmetries; end behavior; and periodicity*. ★ **HSF.IF.B.5:** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★ **HSF.IF.B.6:** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★
Strand	**Algebraic Reasoning - Functions: Interpreting Functions** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.14** Analyze functions using different representations.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSF.IF.C.7:** Graph functions expressed symbolically as well as show and describe key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ **HSF.IF.C.7a:** Graph linear and quadratic functions and show intercepts, maxima, and minima.

	HSF.IF.C.7b: i) Graph square root and piecewise-defined functions, (including step functions and absolute value functions), as well as show and describe key features of the graph. **(+) HSF.IF.C.7b: ii)** Graph cube root functions, as well as show and describe key features of the graph. **(+) HSF.IF.C.7c:** Graph polynomial functions of degree three or higher, identifying zeros when suitable factorizations (in factored form or easily factorable) are available, and showing end behavior. **(+) HSF.IF.C.7d:** Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. **HSF.IF.C.7e: i)** Graph exponential functions, showing intercepts and end behavior, and **(+) HSF.IF.C.7e: ii)** Graph logarithmic functions, showing intercepts and end behavior and trigonometric functions, showing period, midline, and amplitude. **HSF.IF.C.8:** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **HSF.IF.C.8a:** Use the process of factoring and completing the square in a quadratic function to show zeros, maximum and minimum values, and symmetry of the graph, and interpret these in terms of a context. **HSF.IF.C.8b:** Use the properties of exponents to interpret expressions for exponential functions. For example, apply the properties to financial situations such as identifying appreciation and depreciation rate for the value of a house or car sometime after its initial purchase: $V_n = P(1 + r)^n$. **HSF.IF.C.9:** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum*.
Strand	**Algebraic Reasoning - Functions: Building Functions** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.15** Build a function that models a relationship between two quantities. ★
	Adolescence
	Grades 9-Diploma

Performance Expectations	**HSF.BF.A.1:** Write a function that describes a relationship between two quantities. ★ **HSF.BF.A.1a:** Determine an explicit expression, a recursive process, or steps for calculation from a context. **HSF.BF.A.1b:** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model*. **(+) HSF.BF.A.1c:** Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time*. **HSF.BF.A.2:** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★
Strand	**Algebraic Reasoning - Functions: Building Functions** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.16** Build new functions from existing functions.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSF.BF.B.3:** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. **HSF.BF.B.4:** Find inverse functions. **HSF.BF.B.4a:** Solve an equation of the form $f(x) = c$ (where c represents the output value of the function) for a simple function f that has an inverse and write an expression for the inverse. *For example, if* $f(x) = 2x^3$, *then solving* $f(x) = c$ *leads to* $x = (c/2)^{1/3}$, *which is the general formula for finding an input from a specific output, c, for this function.* **(+) HSF.BF.B.4b:** Verify by composition that one function is the inverse of another. **(+) HSF.BF.B.4c:** Read values of an inverse function from a graph or a table, given that the function has an inverse. **(+) HSF.BF.B.4d:** Produce an invertible function from a non-invertible function by restricting the domain.

	(+) HSF.BF.B.5: Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
Strand	**Algebraic Reasoning - Functions: Linear, Quadratic, & Exponential Models** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.17** Construct and compare linear, quadratic, and exponential models and solve problems. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSF.LE.A.1:** Distinguish between situations that can be modeled with linear functions and with exponential functions. ★ **HSF.LE.A.1a:** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. **HSF.LE.A.1b:** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **HSF.LE.A.1c:** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **HSF.LE.A.2:** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ★ **HSF.LE.A.3:** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. ★ **(+) HSF.LE.A.4:** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. ★
Strand	**Algebraic Reasoning - Functions: Linear, Quadratic, & Exponential Models**

	Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).
Standard	**AR.A.18** Interpret expressions for function in terms of the situation they model. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSF.LE.B.5:** Interpret the parameters in a linear or exponential function in terms of a context. ★
Strand	**Algebraic Reasoning - Functions: Trigonometric Functions** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.19** Extend the domain of trigonometric functions using the unit circle.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSF.TF.A.1:** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **HSF.TF.A.2:** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **(+) HSF.TF.A.3:** Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for x, π + x, and 2π - x in terms of their values for x, where x is any real number. **(+) HSF.TF.A.4:** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Strand	**Algebraic Reasoning - Functions: Trigonometric Functions** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**

Standard	**AR.A.20 (+)** Model periodic phenomena with trigonometric functions.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**(+) HSF.TF.B.5:** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★ **(+) HSF.TF.B.6:** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **(+) HSF.TF.B.7:** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★
Strand	**Algebraic Reasoning - Functions: Trigonometric Functions** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.21 (+)** Prove and apply trigonometric identities.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**(+) HSF.TF.C.8:** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. **(+) HSF.TF.C.9:** Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Geometric Reasoning

Geometric reasoning is the use of critical thinking, logical argument and spatial reasoning to solve problems and find new relationships. Students must first have a critical understanding of any underlying assumptions and relationships. This allows them to develop coherent knowledge and apply their reasoning skills. In this K-5 strand, students will develop an understanding of the attributes of two- and three-dimensional shapes and apply this knowledge to real-world problems. Students will also be introduced to the coordinate system.

Students in grades 6-8 work with two- and three-dimensional objects to reason about relationships among shapes. They learn to calculate area, surface area, volume, and circumference using multiple methods including decomposing shapes so that they can develop, justify, and use formulas including the Pythagorean Theorem and its converse. They use scale drawings and informal constructions to gain familiarity with the relationships between angles formed by intersecting lines and transformations.

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more complex definitions and reasoning of proofs. Students make geometric constructions using a variety of technological tools and connect these explorations to reasoning and proofs. Attributes of parallel lines intersected by a transversal are further developed and extended into properties of triangles, quadrilaterals, and regular polygons as well as circles using informal and formal reasoning. Fundamental to the concepts of congruence, similarity, and symmetry are transformations which can preserve distance and angles.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity. The Pythagorean Theorem along with these ratios are fundamental in many real-world and theoretical situations. Correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. Concepts of two- and three-dimensional shapes are explored using algebraic formulas and modeling. Students are encouraged to extend their geometric reasoning through the exploration of trigonometric identities and properties of conic sections.

Strand	Geometric Reasoning - Geometry		
Standard	**GR.C.1** Identify, describe, analyze, compare, create, and compose shapes based on their attributes.		
	Childhood		
	Kindergarten	Grade 1 Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.	Grade 2 Sizes are compared directly or visually, not compared by measuring.
Performance Expectations	**K.G.A.1:** Describe objects in the environment using names of shapes and describe the relative positions of these objects using terms such as *above*, *below*, *beside, in front of*, *behind*, and *next to*. **K.G.A.2:** Correctly name shapes regardless of their orientations or overall size. **K.G.A.3:** Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid"). **K.G.B.4:** Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal	**1.G.A.1:** Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	**2.G.A.1:** Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals (including squares, rectangles, rhombuses, and trapezoids) pentagons, hexagons, and cubes. Sizes are compared directly or visually, not compared by measuring.

	language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). **K.G.B.5:** Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. **K.G.B.6:** Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle*?"	**1.G.A.2:** Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. **1.G.A.3:** Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*.	**2.G.A.2:** Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. **2.G.A.3:** Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two

		Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.
Strand	**Geometric Reasoning - Geometry**		
Standard	**GR.C.2** Analyze, compare, create, and compose shapes based on their attributes.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations	**3.G.A.1:** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. **3.G.A.2:** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each*		**5.G.B.3:** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

	part as 1/4 of the area of the shape.		**5.G.B.4:** Classify two-dimensional figures in a hierarchy based on properties. (e.g., all rectangles are parallelograms, because they are all quadrilaterals with two pairs of opposite sides parallel.)
Strand	**Geometric Reasoning - Geometry**		
Standard	**GR.C.3** Draw and identify lines and angles and classify shapes by properties of their lines and angles.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations		**4.G.A.1:** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. **4.G.A.2:** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category and identify right triangles. **4.G.A.3:** Recognize a line of symmetry for a two-dimensional	

		figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	
Strand	**Geometric Reasoning - Geometry**		
Standard	**GR.C.4** Graph points on the coordinate plane to solve real-world and mathematical problems.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations			**5.G.A.1:** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

			5.G.A.2: Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.

Strand	**Geometric Reasoning - Geometry**
Standard	**GR.EA.1** Solve real-world and mathematical problems involving angle measure, area, surface area, and volume.
	Early Adolescence
	Grades 6-8
Performance Expectations	**6.G.A.1:** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. **6.G.A.2:** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = B h (where B stands for the area of the base) to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **6.G.A.3:** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **6.G.A.4:** Represent three-dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

	7.G.B.4: Know that a circle is a two-dimensional shape created by connecting all the points equidistant from a fixed point called the center of the circle. Understand and describe the relationships among the radius, diameter, circumference and area of a circle. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. **7.G.B.5:** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **7.G.B.6:** Solve real-world and mathematical problems involving area, volume and surface area of two- and/or three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. **8.G.C.9:** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
Strand	**Geometric Reasoning - Geometry**
Standard	**GR.EA.2** Draw, construct, and describe geometrical figures and describe the relationships between them.
	Early Adolescence
	Grades 6-8
Performance Expectations	**7.G.A.1:** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. **7.G.A.2:** Draw (freehand, with ruler and protractor, and with technology) two-dimensional geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **7.G.A.3:** Describe the shape of the cross-section two-dimensional face of the figures that results from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
Strand	**Geometric Reasoning - Geometry**
Standard	**GR.EA.3** Understand congruence and similarity using physical models, transparencies, or geometry software,
	Early Adolescence
	Grades 6-8

Performance Expectations	**8.G.A.1:** Verify experimentally the properties of rotations, reflections, and translations: **8.G.A.1a:** Lines are taken to lines, and line segments to line segments of the same length. **8.G.A.1b:** Angles are taken to angles of the same measure. **8.G.A.1c:** Parallel lines are taken to parallel lines. **8.G.A.2:** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. **8.G.A.3:** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. **8.G.A.4:** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. **8.G.A.5:** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so*.
Strand	**Geometric Reasoning - Geometry**
Standard	**GR.EA.4** Understand and apply the Pythagorean Theorem.
	Early Adolescence
	Grades 6-8
Performance Expectations	**8.G.B.6:** Explain a proof of the Pythagorean Theorem and its converse using pictures, diagrams, narratives or models. **8.G.B.7:** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. **8.G.B.8:** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Strand	**Geometric Reasoning - Geometry: Congruence**
Standard	**GR.A.1** Experiment with transformations in the plane.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSG.CO.A.1:** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. **HSG.CO.A.2:** Represent transformations in the plane using, e.g., transparencies and/or geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). **HSG.CO.A.3:** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. **HSG.CO.A.4:** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. **HSG.CO.A.5:** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
Strand	**Geometric Reasoning - Geometry: Congruence**
Standard	**GR.A.2** Understand congruence in terms of rigid motions.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSG.CO.B.6:** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **HSG.CO.B.7:** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

	HSG.CO.B.8: Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
Strand	**Geometric Reasoning - Geometry: Congruence**
Standard	**GR.A.3** Prove geometric theorems and when appropriate, the converse of theorems.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSG.CO.C.9:** Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints*. **HSG.CO.C.10:** Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point*. **HSG.CO.C.11:** Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals*.
Strand	**Geometric Reasoning - Geometry: Congruence**
Standard	**GR.A.4** Make geometric constructions.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSG.CO.D.12:** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line*. **HSG.CO.D.13:** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
Strand	**Geometric Reasoning - Geometry: Similarity, Right Triangles, & Trigonometry**

Standard	**GR.A.5** Understand similarity in terms of similarity transformations.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSG.SRT.A.1:** Verify experimentally the properties of dilations given by a center and a scale factor: **HSG.SRT.A.1a:** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. **HSG.SRT.A.1b:** The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **HSG.SRT.A.2:** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **HSG.SRT.A.3:** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
Strand	**Geometric Reasoning - Geometry: Similarity, Right Triangles, & Trigonometry**
Standard	**GR.A.6** Prove theorems involving similarity using a variety of ways of writing proofs, showing validity of underlying reasoning.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSG.SRT.B.4:** Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* **HSG.SRT.B.5:** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
Strand	**Geometric Reasoning - Geometry: Similarity, Right Triangles, & Trigonometry** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).**
Standard	**GR.A.7** Define trigonometric ratios and solve problems involving right triangles.
	Adolescence
	Grades 9-Diploma

Performance Expectations	**HSG.SRT.C.6:** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **HSG.SRT.C.7:** Explain and use the relationship between the sine and cosine of complementary angles. **HSG.SRT.C.8:** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. For example, find the current height of the tallest pine tree in Maine using the angle of elevation and the distance from the tree. ★
Strand	**Geometric Reasoning - Geometry: Similarity, Right Triangles, & Trigonometry** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**GR.A.8 (+)** Apply trigonometry to general triangles.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**(+) HSG.SRT.D.9:** Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **(+) HSG.SRT.D.10:** Prove the Laws of Sines and Cosines and use them to solve problems. **(+) HSG.SRT.D.11:** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Strand	**Geometric Reasoning - Geometry: Circle** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**GR.A.9** Understand and apply theorems about circles.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSG.C.A.1:** Prove that all circles are similar.

	HSG.C.A.2: Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* **HSG.C.A.3:** Construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle. **(+) HSG.C.A.4:** Construct a tangent line from a point outside a given circle to the circle.
Strand	**Geometric Reasoning - Geometry: Circle**
Standard	**GR.A.10** Find arc lengths and areas of sectors of circles.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSG.C.B.5:** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
Strand	**Geometric Reasoning - Geometry: Expressing Geometric Properties with Equations** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**GR.A.11** Translate between the geometric description and the equation for a conic section.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSG.GPE.A.1:** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **HSG.GPE.A.2:** Derive the equation of a parabola given a focus and directrix. **(+) HSG.GPE.A.3:** Derive the equations of ellipses and hyperbolas given the foci ~~and directrix~~, using the fact that the sum or difference of distances from the foci is constant.

Strand	**Geometric Reasoning - Geometry: Expressing Geometric Properties with Equations** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).**
Standard	**GR.A.12** Use coordinates to prove simple geometric theorems algebraically.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSG.GPE.B.4:** Use coordinates to prove simple geometric theorems algebraically including the distance formula and its relationship to the Pythagorean Theorem. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* **HSG.GPE.B.5:** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). **HSG.GPE.B.6:** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **HSG.GPE.B.7:** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ★
Strand	**Geometric Reasoning - Geometry: Geometric Measurements & Dimension** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**GR.A.13** Explain volume formulas and use them to solve problems.
	Adolescence
	Grades 9-Diploma

Performance Expectations	**HSG.GMD.A.1:** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and/or informal limit arguments*. **(+) HSG.GMD.A.2:** Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. **HSG.GMD.A.3:** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★
Strand	**Geometric Reasoning - Geometry: Geometric Measurements & Dimension**
Standard	**GR.A.14** Visualize relationships between two-dimensional and three-dimensional objects.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSG.GMD.B.4:** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
Strand	**Geometric Reasoning - Geometry: Modeling with Geometry** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).**
Standard	**GR.A.15** Apply geometric concepts in modeling situations. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSG.MG.A.1:** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★ **HSG.MG.A.2:** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). ★ **HSG.MG.A.3:** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

Statistical Reasoning

Statistical reasoning is the way people analyze data and make sense of information. It involves generalizations that connect one concept to another. In this K-5 strand, students will develop strategies to represent and interpret data, describe and compare measurable attributes, and understand concepts of measurement including perimeter, area, volume, time, and money.

Students in grades 6-8 continue to develop their ability to think statistically. Measures of central tendency (mean, median, and mode) as well as measures of variability (range, interquartile range, mean absolute deviation) are used to describe data. Previous work with single data distributions is expanded to compare two data distributions and address questions about differences between populations. Informal work with random sampling and learning about the importance of representative samples for drawing inferences is introduced. Students then expand their statistical understanding to include connections involving modeling with linear equations, as well as non-linear expressions. Looking for patterns in a bivariate data system is emphasized.

In grades 9-12 students extend their statistical understanding of univariate and bi-variate data in a real-world context. This understanding is used to make decisions or predictions based on the data. Since data can be variable, statistics provide the tools for taking this variability into account. Data can be categorical or quantitative in nature. Appropriate methods for collecting, displaying, summarizing, and analyzing data are learned and employed. Algebraic and geometric reasoning are utilized to create linear regression models in order to interpret the relationship between two quantitative variables when appropriate.

The conditions under which data are collected and the use of randomization in the design of a study are necessary for drawing valid conclusions about the population under study. Since random processes can be described mathematically by using a probability model, the role of probability in making predictions or in making decisions becomes evident. Technology makes it possible to generate plots, find regression functions, compute correlation coefficients, and run simulations to better understand data. Statistical reasoning is a deeply rich and complex process which is essential to comprehend in order to stay informed in civic matters and personal decision-making.

Strand	**Statistical Reasoning - Measurement & Data**		
Standard	**SR.C.1** Describe and compare measurable attributes.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations	**K.MD.A.1:** Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. **K.MD.A.2:** Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter*.	**1.MD.A.1:** Order three objects by length; compare the lengths of two objects indirectly by using a third object. **1.MD.A.2:** Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being*	**2.MD.A.4:** Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. **2.MD.A.1:** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. **2.MD.A.2:** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how

	K.MD.B.3: Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.)	*measured is spanned by a whole number of length units with no gaps or overlaps*.	the two measurements relate to the size of the unit chosen. **2.MD.A.3:** Estimate lengths using units of inches, feet, centimeters, and meters.
Strand	**Statistical Reasoning - Measurement & Data**		
Standard	**SR.C.2** Represent and interpret data.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations		**1.MD.C.4:** Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	**2.MD.D.9:** Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Organize and record data on a line plot, where the horizontal scale is marked off in whole-number units.

			2.MD.D.10: Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.
Strand	**Statistical Reasoning - Measurement & Data**		
Standard	**SR.C.3** Relate addition and subtraction to length.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations			**2.MD.B.5:** Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. **2.MD.B.6:** Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and

			differences within 100 on a number line diagram.
Strand	**Statistical Reasoning - Measurement & Data**		
Standard	**SR.C.4** Work with time and money.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations		**1.MD.B.3:** Tell and write time in hours and half-hours using analog and digital clocks. **1.MD.D.5:** Identify the coins and each corresponding value. (e.g. penny, nickel, dime, and quarter)	**2.MD.C.7:** Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. **2.MD.C.8:** Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?
Strand	**Statistical Reasoning - Measurement & Data**		
Standard	**SR.C.5** Solve problems involving measurement, conversion of measurement and estimation of intervals of time, liquid volumes, and masses of objects.		
	Childhood		
	Grade 3 Excludes compound units such as cm^3 and finding the geometric volume of a container Excludes multiplicative comparison problems (problems involving notions of "times as much")	Grade 4	Grade 5
Performance Expectations	**3.MD.A.1:** Tell and write time to the nearest minute and measure time intervals in minutes using		

	analog and digital clocks. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. **3.MD.A.2:** Measure and estimate liquid volumes and masses of objects using standard metric units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same metric units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.	**4.MD.A.1:** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in.*	**5.MD.A.1:** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

		Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), … **4.MD.A.2:** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	
Strand	**Statistical Reasoning - Measurement & Data**		
Standard	**SR.C.6** Represent and interpret data.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations	**3.MD.B.3:** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For*		

	example, draw a bar graph in which each square in the bar graph might represent 5 pets. **3.MD.B.4:** Generate measurement data by measuring lengths of objects using rulers marked with halves and fourths of an inch. Record and show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or fourths.	**4.MD.B.4:** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection*.	**5.MD.B.2:** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally*.
Strand	**Statistical Reasoning - Measurement & Data**		
Standard	**SR.C.7** Understand concepts of Geometric measurement: involving perimeter, area, and volume.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations	**3.MD.C.5:** Recognize area as an attribute of plane figures and understand concepts of area measurement. **3.MD.C.5a:** A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area,		**5.MD.C.3:** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **5.MD.C.3a:** A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of

	and can be used to measure area. **3.MD.C.5b:** A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. **3.MD.C.6:** Measure areas by counting unit squares (square cm, square m, square in, square ft, and non- standard units **3.MD.C.7:** Relate area to the operations of multiplication and addition. **3.MD.C.7a:** Find the area of a rectangle with whole-number side lengths by tiling it and show that the area is the same as would be found by multiplying the side lengths. **3.MD.C.7b:** Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems and represent whole-number products as rectangular		volume, and can be used to measure volume. **5.MD.C.3b:** A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. **5.MD.C.4:** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and non-standard units. **5.MD.C.5:** Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. When finding volumes of objects answers will be in cubic units. **5.MD.C.5a:** Find the volume of a right rectangular prism with whole-number edge lengths by packing it with unit cubes and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by

	areas in mathematical reasoning. **3.MD.C.7c:** Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning. **3.MD.C.7d:** Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. **3.MD.D.8:** Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the	**4.MD.A.3:** Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor*.	multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. **5.MD.C.5b:** Apply the formulas *V = l × w × h* and *V = B × h (where B stands for the area of the base)* for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. **5.MD.C.5c:** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

	side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.		
Strand	**Statistical Reasoning - Measurement & Data**		
Standard	**SR.C.8** Geometric measurement: understand concept of angle and measure angles.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations		**4.MD.C.5:** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: **4.Md.C.5a:** An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.	

		4.MD.C.5b: An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. **4.MD.C.6:** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **4.MD.C.7:** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	

Strand	**Statistical Reasoning - Statistics & Probability**
Standard	**SR.EA.1** Summarize distribution using measures of center, variability, and graphical displays.
	Early Adolescence
	Grades 6-8
Performance Expectations	**6.SP.A.1:** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages*.

	6.SP.A.2: Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center (mean, median and/or mode), spread (range and/or interquartile range), and overall shape. **6.SP.A.3:** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. **6.SP.B.4:** Display numerical data in plots on a number line, including dot plots, histograms, and box plots. **6.SP.B.5:** Summarize numerical data sets in relation to their context, such as by: **6.SP.B.5a:** Reporting the number of observations. **6.SP.B.5b:** Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. **6.SP.B.5c:** Calculating quantitative measures of center (median and/or mean) and variability (range and/or interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. **6.SP.B.5d:** Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
Strand	**Statistical Reasoning - Statistics & Probability**
Standard	**SR.EA.2** Use random sampling, visual representations, and measures of center and variability to draw inferences about one or more populations.
	Early Adolescence
	Grades 6-8
Performance Expectations	**7.SP.A.1:** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. **7.SP.A.2:** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean length of a largemouth bass in a lake*

	by randomly sampling largemouth bass from the lake; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be. **7.SP.B.3:** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team and both distributions have similar variability (mean absolute deviation) of about 5 cm. The difference between the mean heights of the two teams (10 cm) is about twice the variability (5 cm mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable*. **7.SP.B.4:** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book*.
Strand	**Statistical Reasoning - Statistics & Probability**
Standard	**SR.EA.3** Investigate chance processes and develop, use, and evaluate probability models.
	Early Adolescence
	Grades 6-8
Performance Expectations	**7.SP.C.5:** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. **7.SP.C.6:** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times*. **7.SP.C.7:** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

	7.SP.C.7a: Develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected*. **7.SP.C.7b:** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* **7.SP.C.8:** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. **7.SP.C.8a:** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. **7.SP.C.8b:** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. **7.SP.C.8c:** Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*
Strand	**Statistical Reasoning - Statistics & Probability**
Standard	**SR.EA.4** Investigate patterns of association in bivariate data.
	Early Adolescence
	Grades 6-8
Performance Expectations	**8.SP.A.1:** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. **8.SP.A.2:** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. **8.SP.A.3:** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope*

	of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height. **8.SP.A.4:** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*
Strand	**Statistical Reasoning - Statistics & Probability: Interpreting Categorical & Quantitative Data** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).**
Standard	**SR.A.1** Summarize, represent, and interpret data on a single count or measurement variable. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSS.ID.A.1:** Represent data with plots on the real number line (dot plots, histograms, and box plots). ★ **HSS.ID.A.2:** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. ★ **HSS.ID.A.3:** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★ **HSS.ID.A.4:** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. ★

Strand	**Statistical Reasoning - Statistics & Probability: Interpreting Categorical & Quantitative Data** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).**
Standard	**SR.A.2** Summarize, represent, and interpret data on two categorical variables and two quantitative variables. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSS.ID.B.5:** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. ★ **HSS.ID.B.6:** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ★ **HSS.ID.B.6a:** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. ★ **HSS.ID.B.6b:** Informally assess the fit of a function by plotting and analyzing residuals. ★ **HSS.ID.B.6c:** Fit a linear function for a scatter plot that suggests a linear association. ★
Strand	**Statistical Reasoning - Statistics & Probability: Interpreting Categorical & Quantitative Data** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).**
Standard	**SR.A.3** Interpret linear models. ★
	Adolescence
	Grades 9-Diploma

Performance Expectations	**HSS.ID.C.7:** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ★ **HSS.ID.C.8:** Compute (using technology) and interpret the correlation coefficient of a linear fit. ★ **HSS.ID.C.9:** Distinguish between correlation and causation. ★
Strand	**Statistical Reasoning - Statistics & Probability: Making Inferences & Justifying Conclusions** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).**
Standard	**SR.A.4** Understand and evaluate random processes underlying statistical experiments. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSS.IC.A.1:** Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ★ **HSS.IC.A.2:** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model*? ★
Strand	**Statistical Reasoning - Statistics & Probability: Making Inferences & Justifying Conclusions** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).**
Standard	**SR.A.5** Make inferences and justify conclusions from sample surveys, experiments, and observational studies. ★
	Adolescence

	Grades 9-Diploma
Performance Expectations	**HSS.IC.B.3:** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. ★ **HSS.IC.B.4:** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. ★ **HSS.IC.B.5:** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. ★ **HSS.IC.B.6:** Evaluate reports based on data. *For example, use an article in the local news and interpret the validity of the information presented. Consider animal wildlife reports, medical studies, and/or manufacturer claims.* ★
Strand	**Statistical Reasoning - Statistics & Probability: Conditional Probability & the Rules of Probability** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).**
Standard	**SR.A.6** Understand independence and conditional probability and use them to interpret data. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSS.CP.A.1:** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). ★ **HSS.CP.A.2:** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. ★

	HSS.CP.A.3: Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. ★ **HSS.CP.A.4:** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* ★ **HSS.CP.A.5:** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* ★
Strand	**Statistical Reasoning - Statistics & Probability: Conditional Probability & the Rules of Probability** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**SR.A.7** Use the rules of probability to compute probabilities of compound events in a uniform probability model. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSS.CP.B.6:** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. ★ **HSS.CP.B.7:** Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model. ★

	(+) HSS.CP.B.8: Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. ★ **(+) HSS.CP.B.9:** Use permutations and combinations to compute probabilities of compound events and solve problems. ★
Strand	**Statistical Reasoning - Statistics & Probability: Using Probability to Make Decisions** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**SR.A.8 (+)** Calculate expected values and use them to solve problems. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	**(+) HSS.MD.A.1:** Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. ★ **(+) HSS.MD.A.2:** Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. ★ **(+) HSS.MD.A.3:** Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* ★ **(+) HSS.MD.A.4:** Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on*

	the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households? ★
Strand	**Statistical Reasoning - Statistics & Probability: Using Probability to Make Decisions** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**SR.A.9 (+)** Use probability to evaluate outcomes of decisions. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	**(+) HSS.MD.B.5:** Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. ★ **(+) HSS.MD.B.5a:** Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* ★ **(+) HSS.MD.B.5b:** Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* ★ **(+) HSS.MD.B.6:** Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). ★ **(+) HSS.MD.B.7:** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game and replacing the goalie with an extra skater). ★

Definitions:
Strand: A body of knowledge in a content area identified by a simple title.
Standard: Enduring understandings and skills that students can apply and transfer to contexts that are new to the student.
Performance Expectation: Building blocks to the standard and measurable articulations of what the student understands and can do.