Mathematics Standards Introduction

A strong mathematics education depends upon a clear understanding of its interrelated concepts, skills and practices to ensure students are on the pathway to success in their academic careers. The knowledge and skills students need to be prepared for mathematics in college, career, and life are woven throughout the K-12 mathematics performance expectations.

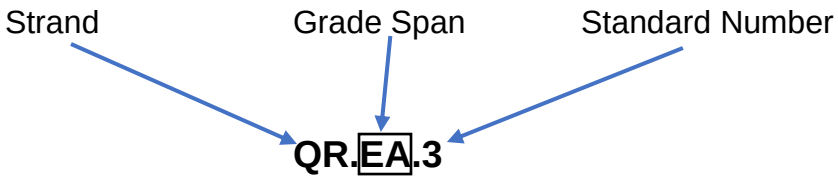
Outline of Mathematics Strands and Standards

These mathematical performance expectations are building blocks to standards. The standards are grouped into four strands:

- **Quantitative Reasoning** (Blue)**:** Counting and Cardinality, Number and Operations in Base Ten, Number and Operations Fractions, Ratio and Proportional Relations, The Number System, and Number and Quantity.
- **Algebraic Reasoning** (Green)**:** Operations and Algebraic Thinking, Expressions and Equations, Functions, and Algebra
- **Geometric Reasoning** (Red)**:** Geometry
- **Statistical Reasoning** (Purple)**:** Measurement and Data, Statistics and Probability

These mathematical performance expectations are broken into three grade spans: **C**hildhood (K-5), **E**arly **A**dolescence (6-8), and **A**dolescence (9-Diploma). The strands are color-coded, as indicated above, for continuity throughout the grade spans. Standards do not work in isolation, they are connected through and across strands.

How to Read the Standards



Within the high school performance expectations, modeling is woven throughout the four strands and is denoted with a star (★). The high school standards also contain some performance expectations which are denoted by a plus (**+**). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.

The Guiding Principles & Standards for Mathematical Practice

The Guiding Principles influence education in Maine and should be reflected throughout Mathematics curriculum. The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Full descriptions of the Guiding Principles and Standards for Mathematical Practice can be found in the Supplemental Material. Examples of how students can show evidence of those **Guiding Principles** and **Standards for Mathematical Practice** **may** include:

Guiding Principles

A. **A clear and effective communicator:** Students will use written, oral, symbolic, and visual forms of expression to communicate mathematically.

B. **A self-directed and lifelong learner:** Students generate and persevere in solving questions while demonstrating a growth mindset.

C. **A creative and practical problem solver:** Students will pose and solve mathematical problems by using a variety of strategies that connect to real-world examples.

D. **A responsible and involved citizen:** Students make sense of the world around them through mathematics including economic literacy.

E. **An integrative and informed thinker:** Students connect mathematics to other learning by understanding the interrelationships of mathematical ideas and the role math plays in other disciplines and life.

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them:** Students will plan strategies to use and persevere in solving math problems.

2. **Reason abstractly and quantitatively:** Students will think about numbers in many ways and make sense of numerical relationships as they solve problems.

3. **Construct viable arguments and critique the reasoning of others:** Students will explain their thinking and make sense of the thinking of others.

4. **Model with mathematics:** Students will use representations to show their thinking in a variety of ways.

5. **Use appropriate tools strategically:** Students will use math tools such as tables, diagrams, and technology to explore and deepen their understanding of concepts.

6. **Attend to precision:** Students will use precise mathematical language and check their work for accuracy.

7. **Look for and make use of structure:** Students will use their current mathematical understandings to identify patterns and structure to make sense of new learning.

8. **Look for and express regularity in repeated reasoning:** Students will look for patterns and rules to help create general methods and shortcuts that can be applied to similar mathematical problems.

Algebraic Reasoning

Algebraic reasoning is about generalizing arithmetic operations and determining unknown quantities by recognizing and analyzing patterns along with developing generalizations about these patterns. In this K-5 strand, students explore, analyze, represent, and generalize mathematical ideas and relationships. Students will develop an understanding of the fundamental properties of number and operations, understand the use of the equal sign to represent equivalence, and use quantitative reasoning to understand mathematical relationships.

Students in grades 6-8 progress in their understanding of variables in mathematical expressions and equations. They understand that expressions in different forms can be equivalent, use the properties of operations to rewrite expressions in equivalent forms, and describe relationships between quantities. Students begin to analyze and solve real-world and mathematical problems using equations and inequalities. They construct and interpret tables and graphs. Understanding builds from writing and solving simple equations to solving proportional situations. These skills lead to exploring slope and y-intercept and relationships between variables, and eventually include multiple equations to solve systems of linear equations. Students grow to understand that the concept of a function is a rule that assigns one output to each input, and they learn to translate among different representations of functions.

In grades 9-12, students will continue to develop their understanding of expressions, equations, functions and function notation. They will interpret the structure of algebraic expressions and be able to write expressions in equivalent forms to reveal information and to solve problems. Students will perform arithmetic operations on polynomials and rewrite rational functions. An understanding of the relationship between zeros and factors of polynomials will transition into using polynomial identities to solve problems. Students will create equations that describe relationships and solve equations as a process of reasoning (with appropriate justification). They will represent and solve equations, inequalities, and systems of equations using a variety of mathematically sound techniques.

Students will interpret functions that arise in applications in terms of context and analyze functions using different representations. They will build functions that model relationships between two quantities, and build new functions from existing functions through transformations, combinations, compositions, and examining the inverse. Students will construct and compare linear, quadratic, and exponential models and use those models to solve problems. They will interpret expressions for functions in terms of the situation they model. Students will be encouraged to extend their understanding of algebra and functions and apply similar processes of reasoning to polynomial, logarithmic and trigonometric functions and their graphs.

Strand	**Algebraic Reasoning - Operations and Algebraic Thinking**		
Standard	**AR.C.1** Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.		
	Childhood		
	Kindergarten (Drawings need not show detail but should show the mathematics in the problem. This applies wherever drawings are mentioned in the Standards.)	Grade 1	Grade 2
Performance Expectations	**K.OA.A.1:** Represent addition and subtraction with objects, fingers, mental images, drawings sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. **K.OA.A.2:** Solve addition and subtraction word problems, and add and subtract within 10, (e.g., by using objects or drawings to represent the problem).	**1.OA.A.1:** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	**2.OA.A.1:** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

	K.OA.A.3: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). **K.OA.A.4:** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. **K.OA.A.5:** Fluently add and subtract within 5 including zero.	**1.OA.A.2:** Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.)	
Strand	**Algebraic Reasoning - Operations and Algebraic Thinking**		

Standard	**AR.C.2** Understand and apply properties of operation and the relationship between addition and subtraction within 20.		
	Childhood		
	Kindergarten	Grade 1 Students need not use formal terms for these properties.	Grade 2
Performance Expectations		**1.OA.B.3:** Apply properties of operations as strategies to add. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.) a + 0 = a (Additive identity property of 0)* **1.OA.B.4:** Understand subtraction as an unknown-addend problem. *For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.* **1.OA.C.5:** Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). **1.OA.C.6:** Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.	**2.OA.B.2:** Fluently add and subtract within 20 using mental strategies. By end of Grade 2,

		Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). **1.OA.D.7:** Understand the meaning of the equal sign and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2. **1.OA.D.8:** Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ - 3, 6 + 6 = _.*	know from memory all sums of two one-digit numbers.
Strand	**Algebraic Reasoning - Operations and Algebraic Thinking**		

Standard	**AR.C.3** Work with equal groups of objects to gain foundations for multiplication.		
	Childhood		
	Kindergarten	Grade 1	Grade 2
Performance Expectations			**2.OA.C.3:** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. **2.OA.C.4:** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
Strand	**Algebraic Reasoning - Operations and Algebraic Thinking**		
Standard	**AR.C.4** Understand properties of multiplication and the relationship between multiplication and division to represent and solve problems within 100.		
	Childhood		
	Grade 3 Students need not use formal terms for these properties.	Grade 4	Grade 5
Performance Expectations	**3.OA.A.1:** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5		

	groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7*. **3.OA.A.2:** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8*. **3.OA.A.3:** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.		

	3.OA.A.4: Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?* **3.OA.B.5:** Apply properties of operations as strategies to multiply. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* **3.OA.B.6:** Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the*		

	number that makes 32 when multiplied by 8. **3.OA.C.7:** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.		
Strand	**Algebraic Reasoning - Operations and Algebraic Thinking**		
Standard	**AR.C.5** Solve problems involving the four operations.		
	Childhood		
	Grade 3 This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in conventional order when there are no parentheses to specify a particular order (Order of Operations).	Grade 4	Grade 5
Performance Expectations		**4.OA.A.1:** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	

	3.OA.D.8: Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	**4.OA.A.2:** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. **4.OA.A.3:** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	
Strand	**Algebraic Reasoning - Operations and Algebraic Thinking**		
Standard	**AR.C.6** Gain familiarity with factors and multiples.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations		**4.OA.B.4:** Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is	

		a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	
Strand	**Algebraic Reasoning - Operations and Algebraic Thinking**		
Standard	**AR.C.7** Write and interpret numerical expressions.		
	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations			**5.OA.A.1:** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. **5.OA.A.2:** Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*
Strand	**Algebraic Reasoning - Operations and Algebraic Thinking**		
Standard	**AR.C.8** Identify, explain, generate and analyze patterns.		

	Childhood		
	Grade 3	Grade 4	Grade 5
Performance Expectations	**3.OA.D.9:** Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends*.	**4.OA.C.5:** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way*.	**5.OA.B.3:** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so*.

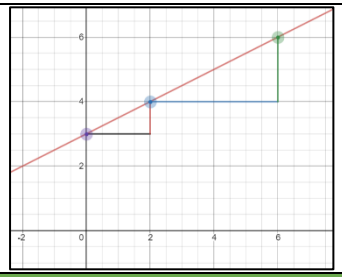
Strand	**Algebraic Reasoning - Expressions and Equations**
Standard	**AR.EA.1** Apply and extend previous understandings of arithmetic to algebraic expressions.
	Early Adolescence
	Grades 6-8
Performance Expectations	**6.EE.A.1:** Write and evaluate numerical expressions involving whole-number exponents. **6.EE.A.2:** Write, read, and evaluate expressions in which letters represent numbers. **6.EE.A.2a:** Write expressions that record operations with numbers and with letters representing numbers. *For example, express the calculation "Subtract y from 5" as 5 - y*.

	6.EE.A.2b: Identify parts of an expression using mathematical terms (including but not limited to: sum, term, product, factor, quotient, coefficient, variable, constant); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (x + 7) as a product of two factors; view (x + 7) as both a single entity and a sum of two terms*. **6.EE.A.2c:** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, using the order of operations. *For example, use the formulas $V = s^3$ and $A = 6\ s^2$ to find the volume and surface area of a cube with sides of length s = 1/2*. **6.EE.A.3:** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to factor the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y*. **6.EE.A.4:** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*
Strand	**Algebraic Reasoning - Expressions and Equations**
Standard	**AR.EA.2** Reason about and solve one-variable equations and inequalities.
	Early Adolescence
	Grades 6-8
Performance Expectations	**6.EE.B.5:** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **6.EE.B.6:** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.EE.B.7:** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. *For example, Sal is paid $0.50*

	per pound of blueberries that she rakes. If she rakes x pounds, and earns $17.25, write and solve an equation that determines how many pounds she raked. **6.EE.B.8:** Write an inequality of the form *x* > *c* or *x* < *c* to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form *x* > *c* or *x* < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
Strand	**Algebraic Reasoning - Expressions and Equations**
Standard	**AR.EA.3** Represent and analyze quantitative relationships between dependent and independent variables.
	Early Adolescence
	Grades 6-8
Performance Expectations	**6.EE.C.9:** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*
Strand	**Algebraic Reasoning - Expressions and Equations**
Standard	**AR.EA.4** Use properties of operations to generate equivalent expressions.
	Early Adolescence
	Grades 6-8
Performance Expectations	**7.EE.A.1:** Apply properties of operations to add, subtract, factor, and expand linear expressions with rational coefficients. *For example, 4x + 2 = 2(2x+1) and -3(x-5/3) = -3x +5* **7.EE.A.2:** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, A shirt is on sale for 20% off the regular price, p. The discount can be expressed as 0.2p. The new price for the shirt can be expressed as p – 0.2p or 0.8p.*
Strand	**Algebraic Reasoning - Expressions and Equations**

Standard	**AR.EA.5** Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
	Early Adolescence
	Grades 6-8
Performance Expectations	**7.EE.B.3:** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation*. **7.EE.B.4:** Use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities. **7.EE.B.4a:** Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* **7.EE.B.4b:** Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make and describe the solutions*.
Strand	**Algebraic Reasoning - Expressions and Equations**
Standard	**AR.EA.6** Work with radicals and integer exponents.
	Early Adolescence
	Grades 6-8
Performance Expectations	**8.EE.A.1:** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = (1/3)^3 = 1/27$.*

	8.EE.A.2: Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. **8.EE.A.3:** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger*. **8.EE.A.4:** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
Strand	**Algebraic Reasoning - Expressions and Equations**
Standard	**AR.EA.7** Understand the connections between proportional relationships, lines, and linear equations.
	Early Adolescence
	Grades 6-8
Performance Expectations	**8.EE.B.5:** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* **8.EE.B.6:** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. *For example, given the line $y = 0.5x + 3$ explain why the similar triangles have the same slope.*

	
Strand	**Algebraic Reasoning - Expressions and Equations**
Standard	**AR.EA.8** Analyze and solve linear equations and pairs of simultaneous linear equations.
	Early Adolescence
	Grades 6-8
Performance Expectations	**8.EE.C.7:** Solve linear equations in one variable. **8.EE.C.7a:** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). **8.EE.C.7b:** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. **8.EE.C.8:** Analyze and solve pairs of simultaneous linear equations. **8.EE.C.8a:** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. **8.EE.C.8b:** Solve systems of two linear equations in two variables algebraically (i.e. by substitution or elimination) and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6*. **8.EE.C.8c:** Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair*.
Strand	**Algebraic Reasoning - Functions**

Standard	**AR.EA.9** Define, evaluate, and compare functions in order to model relationships between quantities.
	Early Adolescence
	Grades 6-8 Function notation is not required for Grade 8.
Performance Expectations	**8.F.A.1:** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. **8.F.A.2:** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change*. **8.F.A.3:** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line*. **8.F.B.4:** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **8.F.B.5:** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Strand	**Algebraic Reasoning - Algebra: Seeing Structure in Expressions** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).**

Standard	**AR.A.1** Interpret the structure of expressions.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSA.SSE.A.1:** Interpret expressions that represent a quantity in terms of its context. ★ **SSE.A.1a:** Interpret parts of an expression, such as terms, factors, and coefficients. ★ **SSE.A.1b:** Interpret multi-part expressions by viewing one or more of their parts as a single entity. *For example, view $P(1+r)^n$ as the product of P and a factor not depending on P and interpret the parts.* ★ **HSA.SSE.A.2:** Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, allowing for it to be recognized as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*
Strand	**Algebraic Reasoning - Algebra: Seeing Structure in Expressions** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).**
Standard	**AR.A.2** Write expressions in equivalent forms to reveal information and to solve problems. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSA.SSE.B.3:** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★ **HSA.SSE.B.3a:** Rewrite a quadratic expression (such as by factoring) to reveal the zeros of the function it defines. ★ **HSA.SSE.B.3b:** Rewrite a quadratic expression (such as by completing the square) to reveal the maximum or minimum value of the function it defines. ★ **HSA.SSE.B.3c:** Use the properties of exponents to transform expressions for exponential functions. *For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* ★ **HSA.SSE.B.4:** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, Watermilfoil in one Maine lake triples in the number of plants each week during the summer when boat propellers are not cleared when exiting the lake. If the lake has 20*

	plants at the beginning of the season, how many plants will exist at the end of the 12-week summer season? What is the general formula for Watermilfoil growth for this lake? ★
Strand	**Algebraic Reasoning - Algebra: Arithmetic with Polynomials & Rational Expressions**
Standard	**AR.A.3** Perform arithmetic operations on polynomials.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSA.APR.A.1:** Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. **HSA.APR.A.1a**: Perform operations on polynomial expressions (addition, subtraction, multiplication, and division), and compare the system of polynomials to the system of integers. **HSA.APR.A.1b**: Factor and/or expand polynomial expressions, identify and combine like terms, and apply the Distributive Property.
Strand	**Algebraic Reasoning - Algebra: Arithmetic with Polynomials & Rational Expressions** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.4** Understand the relationship between zeros and factors of polynomials.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSA.APR.B.2:** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. For example, consider the polynomial function $P(x) = x^4 – 2x^3 + ax^2 + 8x + 12$, where a is an unknown real number. If (x-3) is a factor of this polynomial, what is the value of a? **(+) HSA.APR.B.3:** Identify zeros of polynomials of degree three or higher when suitable factorizations (in factored form or easily factorable) are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Strand	**Algebraic Reasoning - Algebra: Arithmetic with Polynomials & Rational Expressions**

	The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.
Standard	**AR.A.5 (+)** Use polynomial identities to solve problems.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**(+) HSA.APR.C.4:** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.* **(+) HSA.APR.C.5:** Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.
Strand	**Algebraic Reasoning - Algebra: Arithmetic with Polynomials & Rational Expressions** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.6** Rewrite rational expressions.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSA.APR.D.6:** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. **(+) HSA.APR.D.7:** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
Strand	**Algebraic Reasoning -Algebra: Creating Equations and/or Inequalities** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).**
Standard	**AR.A.7** Create equations and/or inequalities that describe numbers or relationships. ★

	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSA.CED.A.1:** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*. ★ **HSA.CED.A.2:** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★ **HSA.CED.A.3:** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods such as lobsters, blueberries, and potatoes.* ★ **HSA.CED.A.4:** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R*. ★
Strand	**Algebraic Reasoning - Algebra: Reasoning with Equations & Inequalities**
Standard	**AR.A.8** Understand solving equations as a process of reasoning and explain the reasoning.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSA.REI.A.1:** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method. **HSA.REI.A.2:** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
Strand	**Algebraic Reasoning - Algebra: Reasoning with Equations & Inequalities** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.9** Solve equations and inequalities in one variable.
	Adolescence
	Grades 9-Diploma

Performance Expectations	**HSA.REI.B.3:** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **HSA.REI.B.4:** Solve quadratic equations in one variable. **HSA.REI.B.4a:** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. **HSA.REI.B.4b: i)** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. **(+) HSA.REI.B.4b: ii)** Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
Strand	**Algebraic Reasoning - Algebra: Reasoning with Equations & Inequalities** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.10** Solve systems of equations.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSA.REI.C.5:** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. **HSA.REI.C.6:** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **HSA.REI.C.7:** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the point(s) of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.* **(+) HSA.REI.C.8:** Represent a system of linear equations as a single matrix equation in a vector variable. **(+) HSA.REI.C.9:** Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).
Strand	**Algebraic Reasoning - Algebra: Reasoning with Equations & Inequalities**

	Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).
Standard	**AR.A.11** Represent and solve equations and inequalities graphically.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSA.REI.D.10:** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation. **HSA.REI.D.11:** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★ **HSA.REI.D.12:** Graph the solutions of a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set of a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
Strand	**Algebraic Reasoning - Functions: Interpreting Functions**
Standard	**AR.A.12** Understand the concept of a function and use function notation.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSF.IF.A.1:** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$. **HSF.IF.A.2:** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **HSF.IF.A.3:** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1*.

Strand	**Algebraic Reasoning - Functions: Interpreting Functions** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).**
Standard	**AR.A.13** Interpret functions that arise in applications in terms of the context. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSF.IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features may include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative and absolute maximums and minimums; symmetries; end behavior; and periodicity*. ★ **HSF.IF.B.5:** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★ **HSF.IF.B.6:** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★
Strand	**Algebraic Reasoning - Functions: Interpreting Functions** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.14** Analyze functions using different representations.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSF.IF.C.7:** Graph functions expressed symbolically as well as show and describe key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ **HSF.IF.C.7a:** Graph linear and quadratic functions and show intercepts, maxima, and minima.

	HSF.IF.C.7b: i) Graph square root and piecewise-defined functions, (including step functions and absolute value functions), as well as show and describe key features of the graph. **(+) HSF.IF.C.7b: ii)** Graph cube root functions, as well as show and describe key features of the graph. **(+) HSF.IF.C.7c:** Graph polynomial functions of degree three or higher, identifying zeros when suitable factorizations (in factored form or easily factorable) are available, and showing end behavior. **(+) HSF.IF.C.7d:** Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. **HSF.IF.C.7e: i)** Graph exponential functions, showing intercepts and end behavior, and **(+) HSF.IF.C.7e: ii)** Graph logarithmic functions, showing intercepts and end behavior and trigonometric functions, showing period, midline, and amplitude. **HSF.IF.C.8:** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **HSF.IF.C.8a:** Use the process of factoring and completing the square in a quadratic function to show zeros, maximum and minimum values, and symmetry of the graph, and interpret these in terms of a context. **HSF.IF.C.8b:** Use the properties of exponents to interpret expressions for exponential functions. For example, apply the properties to financial situations such as identifying appreciation and depreciation rate for the value of a house or car sometime after its initial purchase: $V_n = P(1 + r)^n$. **HSF.IF.C.9:** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum*.
Strand	**Algebraic Reasoning - Functions: Building Functions** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (★).** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.15** Build a function that models a relationship between two quantities. ★
	Adolescence
	Grades 9-Diploma

Performance Expectations	**HSF.BF.A.1:** Write a function that describes a relationship between two quantities. ★ **HSF.BF.A.1a:** Determine an explicit expression, a recursive process, or steps for calculation from a context. **HSF.BF.A.1b:** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model*. **(+) HSF.BF.A.1c:** Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time*. **HSF.BF.A.2:** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★
Strand	**Algebraic Reasoning - Functions: Building Functions** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.16** Build new functions from existing functions.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSF.BF.B.3:** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Okay as written. **HSF.BF.B.4:** Find inverse functions. **HSF.BF.B.4a:** Solve an equation of the form $f(x) = c$ (where c represents the output value of the function) for a simple function f that has an inverse and write an expression for the inverse. *For example, if* $f(x) = 2x^3$, *then solving* $f(x) = c$ *leads to* $x = (c/2)^{1/3}$, *which is the general formula for finding an input from a specific output, c, for this function.* **(+) HSF.BF.B.4b:** Verify by composition that one function is the inverse of another. **(+) HSF.BF.B.4c:** Read values of an inverse function from a graph or a table, given that the function has an inverse. **(+) HSF.BF.B.4d:** Produce an invertible function from a non-invertible function by restricting the domain.

	(+) HSF.BF.B.5: Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
Strand	**Algebraic Reasoning - Functions: Linear, Quadratic, & Exponential Models** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.17** Construct and compare linear, quadratic, and exponential models and solve problems. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSF.LE.A.1:** Distinguish between situations that can be modeled with linear functions and with exponential functions. ★ **HSF.LE.A.1a:** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. **HSF.LE.A.1b:** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **HSF.LE.A.1c:** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **HSF.LE.A.2:** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ★ **HSF.LE.A.3:** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. ★ **(+) HSF.LE.A.4:** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. ★
Strand	**Algebraic Reasoning - Functions: Linear, Quadratic, & Exponential Models**

	Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).
Standard	**AR.A.18** Interpret expressions for function in terms of the situation they model. ★
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSF.LE.B.5:** Interpret the parameters in a linear or exponential function in terms of a context. ★
Strand	**Algebraic Reasoning - Functions: Trigonometric Functions** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.19** Extend the domain of trigonometric functions using the unit circle.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**HSF.TF.A.1:** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **HSF.TF.A.2:** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **(+) HSF.TF.A.3:** Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for x, π + x, and 2π - x in terms of their values for x, where x is any real number. **(+) HSF.TF.A.4:** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Strand	**Algebraic Reasoning - Functions: Trigonometric Functions** **Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appears throughout the high school standards indicated by a star symbol (⋆).** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**

Standard	**AR.A.20 (+)** Model periodic phenomena with trigonometric functions.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**(+) HSF.TF.B.5:** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★ **(+) HSF.TF.B.6:** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **(+) HSF.TF.B.7:** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★
Strand	**Algebraic Reasoning - Functions: Trigonometric Functions** **The high school standards also contain some performance expectations which are denoted by a plus (+). These performance expectations are intended to be extensions of learning. All students should be given opportunities to explore this content, but mastery is not expected.**
Standard	**AR.A.21 (+)** Prove and apply trigonometric identities.
	Adolescence
	Grades 9-Diploma
Performance Expectations	**(+) HSF.TF.C.8:** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. **(+) HSF.TF.C.9:** Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Definitions:

Strand: A body of knowledge in a content area identified by a simple title.

Standard: Enduring understandings and skills that students can apply and transfer to contexts that are new to the student.

Performance Expectation: Building blocks to the standard and measurable articulations of what the student understands and can do.